A LOW COST METHOD TO SENSE VIBRATION USING LASER DISTANCE SENSOR

LIM CHIN CHUAN

PROGRAMME PHYSICS WITH ELECTRONICS
FACULTY OF SCIENCE AND NATURAL RESOURCES
UNIVERSITY MALAYSIA SABAH
2014
**JUDUL:** A low cut method to sense vibration among cars

**IJAZAH:** SARJANA MUDA SAINI DENGAN KEPUSAN PROGRAM FIZIK DENGAN ELEKTRONIK

**SAYA:** LIM CHIN CHUAN (HURUF BESAR)

**SESU PENGAJIAN:** 2015

**Mengaku membenarkan tesis** *(LPSM/SarjanaiDoktor Falsafah)* ini disimpan di Perpustakaan Universiti Malaysia Sabah dengan syarat-syarat kegunaan seperti berikut:-

1. Tesis adalah hakmil ik Universiti Malaysia Sabah.
2. Perpustakaan Universiti Malaysia Sabah dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan tesis ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. Sila tandakan (/) SULIT (Mengandungi maklumat yang berda~ah keselamatan atau kepentingan Malaysia seperti yang termaktub di AKTA RAHSIA RASMI 1972)
   - TIDAK TERHAD
   - TERHAD (Mengandungi maklumat TERHAD yang telah ditentukan oleh organisasi/badan di mana Penyelidikan dijalankan)

**PERPUSTAKAAN UNIVERSITI MALAYSIA SABAH**

Disahkan: NORULAIN BINTI ISMAIL

(TANDATANGAN PENULIS)

(TANDATANGAN POSTAKAWAN)

Alamat tetap: 41,66 Jl/11, Taman Selayang Baru, 75350, Meleka

Nama Penyelidik: Dr. Jedol Airiyou

Tarikh: 19/6/15

Catatan:
- *Potong yang tidak berkenaan.
- *Jika tesis ini SULIT atau TERHAD, sila lampirkan surat daripada pihak berkuasa/organisasi berkenaan dengan menyatakan sekali sebab dan tempoh tesis ini perlu diklasakan sebagai SULIT dan TERHAD.
- *Tesis dimaksudkkan sebagai tesis bagi Ijazah Doktor Falsafah dgn Sarjana Secara penelitian atau disertai bagi pengajian secara kerja kursus dan Laporan Projek Sarjana Muda (LPSM)*
NAME : LIM CHIN CHUAN
MATRIC NUMBER : BS12110300
TITLE : A LOW COST METHOD TO SENSE VIBRATION USING LASER DISTANCE SENSOR
DEGREE : BACHELOR OF SCIENCE (PHYSICS WITH ELECTRONICS)
DATE OF VIVA : 25 MAY 2015

VERIFIED BY

SUPERVISOR
ASSOCIATE PROFESSOR DR. JEDOL DAYOU

EXAMINER
DR. AFISHAH ALIAS

DEAN
PROFESSOR DR. BABA MUSTA
DECLARATION

I hereby declare that the material in this thesis is my own except for quotations, excerpts, equations, summaries and references, which have been duly acknowledged.

2 JUNE 2015

Lim Chin Chuan
BS12110300
ACKNOWLEDGEMENT

I would like to express my deepest gratitude and appreciation to my supervisor, Assoc. Prof. Dr. Jedol Dayou for all his advices, guidance and support in this research work that lead to the completion of this thesis. I wish to thank Dr. Jedol for his wholehearted supervision and continuous effort in showing ways for completing this project. I also wish to give credits to Dr. Chee Fuei Pien for her tireless assistance, provided me with supports unconditionally in terms of experience, ideas and experimental tools. Lastly, I wish to thank Faculty of Science and Natural Resources University Malaysia Sabah for the equipment and facilities that enable the succeed of this project.

LIM CHIN CHUAN
2 JUNE 2015
ABSTRACT

This thesis reports a non-contact method of measuring the relative amplitude, natural frequency and damping ratio of vibration. The low cost model presented in this thesis exploits simple design with sufficient high sensitivity, that make it suitable for practical use. Laser approach is discussed in this thesis for the detection of vibration - by using a laser diode and photo detector. The laser beam was pointed at the reflecting surface of a vibrating loud speaker, with frequency controlled by a function generator. The vibrating loud speaker caused changes in the light intensity that received by the photo detector, thus output voltage of different values. Voltage reading was performed by using an Arduino nano board, while the relative amplitude, natural frequency and damping ratio of the vibration was computed by ActionScript 3.0 in Flash. Displacement sensitivity of the sensor is in the range of millimeters and centimeters, while the vibrating frequency range is below 50Hz. The sensor has shown good accuracy when the vibration frequencies are below 10Hz. However the system is observed to have difficulties in detecting higher frequencies of vibration.
ABSTRAK

Kaedah Kos Rendah untuk Mengesan Getaran dengan Menggunakan Laser

LIST OF CONTENTS

Page

Title iii
Declaration iv
Acknowledgement v
Abstract vi
Abstract vi
List of Contents vii
List of Figures ix
List of Tables x
List of Symbol xi
List of Abbreviations xiii
Chapter 1: Introduction 1
1.1 Introduction 1
1.2 Problem Statement 2
1.3 Objective 2
1.4 Hypothesis 2
1.5 Summary 3
Chapter 2: Theoretical Background 4
2.1 Vibration Physics 4
  2.1.1 Underdamped System 5
  2.1.2 Critically Damped System 6
  2.1.3 Overdamped System 7
  2.1.4 Logarithmic decrement 8
2.2 Interferometry 8
  2.2.1 The Stationary Michelson Interferometer 8
  2.2.2 Changing Path Lengths in the Interferometer 9
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.3 Optimizing Intensity Variation</td>
<td>10</td>
</tr>
<tr>
<td>2.3 Sampling</td>
<td>11</td>
</tr>
<tr>
<td>2.4 Laser Diode</td>
<td>12</td>
</tr>
<tr>
<td>2.4.1 Activation Energy</td>
<td>12</td>
</tr>
<tr>
<td>2.5 Photodiode</td>
<td>13</td>
</tr>
<tr>
<td>2.5.1 pN Junction Photodiode</td>
<td>13</td>
</tr>
<tr>
<td>2.5.2 PIN Photodiode</td>
<td>14</td>
</tr>
<tr>
<td>2.5.3 Avalanche Photodiode (APD)</td>
<td>14</td>
</tr>
<tr>
<td>2.6 Analog-to-Digital Converter (ADC)</td>
<td>14</td>
</tr>
<tr>
<td>2.6.1 Arduino Board Nano</td>
<td>15</td>
</tr>
<tr>
<td>2.7 Software</td>
<td>16</td>
</tr>
<tr>
<td>2.7.1 ActionScript 3.0</td>
<td>16</td>
</tr>
<tr>
<td>2.7.2 Firmata Library</td>
<td>16</td>
</tr>
<tr>
<td>2.7.3 Serproxy</td>
<td>16</td>
</tr>
<tr>
<td><strong>CHAPTER 3: METHODOLOGY</strong></td>
<td>17</td>
</tr>
<tr>
<td>3.1 Measurement</td>
<td>17</td>
</tr>
<tr>
<td>3.2 Materials</td>
<td>17</td>
</tr>
<tr>
<td>3.3 Setup and Procedure</td>
<td>17</td>
</tr>
<tr>
<td><strong>CHAPTER 4: RESULTS</strong></td>
<td>20</td>
</tr>
<tr>
<td>4.1 ActionScript Code</td>
<td>20</td>
</tr>
<tr>
<td>4.2 Readings</td>
<td>27</td>
</tr>
<tr>
<td>4.3 Discussion</td>
<td>29</td>
</tr>
<tr>
<td><strong>CHAPTER 5: CONCLUSION AND DISCUSSION</strong></td>
<td>30</td>
</tr>
<tr>
<td>5.1 Conclusion</td>
<td>30</td>
</tr>
<tr>
<td><strong>REFERENCES</strong></td>
<td>31</td>
</tr>
<tr>
<td><strong>APPENDIX</strong></td>
<td>31</td>
</tr>
<tr>
<td>Appendix A ActionScript Code</td>
<td>31</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Damped system with force diagram</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Displacement Time History of an Underdamped SDOF System</td>
<td>6</td>
</tr>
<tr>
<td>2.3</td>
<td>Response of damped free vibration</td>
<td>1</td>
</tr>
<tr>
<td>2.4</td>
<td>Schematic of the Michelson interferometer.</td>
<td>8</td>
</tr>
<tr>
<td>2.6</td>
<td>Arduino Nano Pin Layout</td>
<td>15</td>
</tr>
<tr>
<td>3.1</td>
<td>Connection of laser distance sensor to the Arduino board</td>
<td>18</td>
</tr>
<tr>
<td>3.2</td>
<td>Flowchart of procedure</td>
<td>19</td>
</tr>
<tr>
<td>4.1</td>
<td>Natural frequencies at preset frequencies</td>
<td>24</td>
</tr>
<tr>
<td>4.2</td>
<td>Relative amplitudes at preset frequencies</td>
<td>25</td>
</tr>
<tr>
<td>4.3</td>
<td>Damping ratios at preset frequencies</td>
<td>25</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 4.1 Waveform of vibration at preset frequencies .......................... 27
Table 4.2 Damping ratios, natural frequencies and relative amplitudes of preset frequencies ............................................................. 29
LIST OF SYMBOL

\( c_c \) Critical damping
\( \delta \) Logarithmic Decrement
\( e_a \) Activation Energy
\( f_c \) Energy Dissipation
\( t_f \) Time to Failure
\( v_0 \) Initial Velocity
\( \dot{x} \) First Derivative of \( x \)
\( \ddot{x} \) Second Derivative of \( x \)
\( x_0 \) Initial Displacement
\( \omega_0 \) Natural Frequency
\( \omega_d \) Damped Circular Natural Frequency
\( \zeta \) Damping Ratio
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Scaling factor</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>i/j</td>
<td>Imaginary Number</td>
</tr>
<tr>
<td>k</td>
<td>Boltzmann’s Constant/ Spring Constant</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>ACC</td>
<td>Automatic Current Control</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>APC</td>
<td>Automatic Power Control</td>
</tr>
<tr>
<td>APD</td>
<td>Avalanche Photodiode</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitted Diode</td>
</tr>
<tr>
<td>MTTF</td>
<td>Median Time To Failure</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Introduction

Vibration measurements are important and being applied extensively especially in engineering industry (e.g., to find out the machine condition) and even in household applications (e.g., monitoring the vibration level of rotating equipment such as fans). Engineers and scientists have designed various type of vibration sensors that can be classified into two major categories: the necessity of the sensor to be in physical contact with the vibrating object or not. The non-contact type vibration sensors have several advantages over the contact type sensors such as the ability to measure vibration of very small objects and do not perturb vibration. Thus, these sensors are usually more favorable.

Everything vibrates with it natural frequency, which is the tendency for that particular system to oscillate without the presence of any external force. By applying similar concept, any desired pattern of vibration can be made by manipulating the natural frequency. A set of factors affecting the natural frequency such as tightness, length, or weight of an object where altering any of these of an object would change the size, inertia, or forces in the system. Meanwhile, knowing the damping ratio of a vibration is also essential. Without considering damping in a building can lead to collapsing by merely wind blows. A car without damping will continue bouncing vigorously after gone through a bumper. Thus it is very important to understand the
damping ratio of a system so that a desired amount of vibration can be controlled.

1.2 Problem Statement
Even as the sensors available in the market made excellent vibration detectors, their price range is considered extortionate to a relatively large group of people. Studies have always been focusing on industrial application in which the sensors are integrated with a bulk of functions and abilities which results in the increasing of the marketing prices of the end products. Taking a laser vibrometer as an example, the design is complicated and pricey. For practical uses - such as household applications, an inexpensive vibration sensing is desirable. Hence a simple vibration sensing model which is generally affordable should be made available.

1.3 Objective
The objective of the study is divided into two major parts as listed.
- To develop a method for the detection of relative amplitudes and natural frequencies, and damping ratio of a vibration by using laser approach.
- To compile the method into a standalone software with the ability to run on modern computers.

1.4 Hypothesis
The laser distance sensor can be used to detect vibration characteristics based on the change in intensity of the reflected laser beam from a reflecting surface that received by the photo detector.

1.5 Summary
Vibration sensing could come in handy for specific purposes, a low cost method is especially preferable with sufficient sensitivity. Analog vibration sensors available in the market are usually for advance operations and thus expensive, while digital vibration sensor can be made for all practical uses with a small cost.
CHAPTER 2

THEORETICAL BACKGROUND

2.1 Vibration Physics

The sample objects to be studied are assumed to have single degree of freedom, which only vibrate at vertical direction.

For balancing the forces on the mass,

\[ F_k + F_c + F_e = F_r \]  \hspace{1cm} (2.1)

However, after the excitation force is released at initial time \( t=0 \), \( F_e = 0 \), \( F_k = -kx(t) \), and \( F_r = ma \). Consider the system undergoes viscous damping, which is the most common type of damping, the energy dissipation in the system is proportional to the velocity of the mass in motion, given by \( f_c = cx(t) \). Equation above can now be written as
\[-c\ddot{x}(t) - kx(t) = m\dddot{x}(t) \hspace{1cm} (2.2)\]

or

\[\ddot{x}(t) + \frac{c}{m}\dot{x}(t) + \frac{k}{m}x(t) = 0 \hspace{1cm} (2.3)\]

To solve this differential equation, assume a solution of the form

\[x(t) = Ae^s \hspace{1cm} (2.4)\]

Take the first and second derivative with respect to time of Equation (2.3),

\[\dot{x}(t) = sAe^s \hspace{1cm} (2.5)\]

\[\ddot{x}(t) = s^2 Ae^s \hspace{1cm} (2.6)\]

Substitute these two equations into equation (2.3) giving

\[s^2 Ae^s + \frac{c}{m}sAe^s + \frac{k}{m}Ae^s = 0 \hspace{1cm} (2.7)\]

Which can be simplified into

\[s^2 + \frac{c}{m}s + \frac{k}{m} = 0 \hspace{1cm} (2.8)\]

By taking quadratic equation formula

\[s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \hspace{1cm} (2.9)\]

\(s\) produces two possible roots

\[s_{1,2} = \frac{-c}{2m} \pm \frac{\sqrt{c^2 - 4km}}{2m} \hspace{1cm} (2.10)\]

Thus the solution is given by

\[x(t) = Ae^{s_1t} + Be^{s_2t} \hspace{1cm} (2.11)\]

Critical damping \(c_c\), and damping ratios \(\zeta\) are introduced. Where

\[c_c = 2m\omega_0 = 2\sqrt{km} \hspace{1cm} (2.12)\]

and

\[\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_0} = \frac{c}{2\sqrt{km}} \hspace{1cm} (2.13)\]

After rearranging equation (2.13), yields
\[
\frac{c}{2m} = \omega_0 \zeta
\] (2.14)

Which is then substitute into equation (2.10) to get

\[
s_{1,2} = -\zeta \omega_0 \pm i \omega_0 \sqrt{1 - \zeta^2}
\] (2.15)

Substitute into equation (2.11) to get

\[
x(t) = e^{-\zeta \omega_0 t} \left( Ae^{i \omega_0 t \sqrt{1 - \zeta^2}} + Be^{-i \omega_0 t \sqrt{1 - \zeta^2}} \right)
\] (2.16)

Applying the initial conditions as \(x(0)=x_0\) (initial displacement) and \(v(0)=v_0\) (initial velocity), \(x_0 = A + B\), substituting these expressions into equation (2.16) and its first derivative gives

\[
x(t) = e^{-\zeta \omega_0 t} \left[ \frac{x_0}{2} \left( \frac{i(v_0 - \zeta \omega_0 x_0)}{2 \omega_0 \sqrt{1 - \zeta^2}} \right) e^{(-i \omega_0 t \sqrt{1 - \zeta^2})} + \frac{x_0}{2} \left( \frac{v_0 - \zeta \omega_0 x_0}{2 \omega_0 \sqrt{1 - \zeta^2}} \right) e^{(i \omega_0 t \sqrt{1 - \zeta^2})} \right]
\] (2.17)

There are three different solutions for the three different values of \(\zeta\) underdamped system where \(0<\zeta<1\); critically damped system where \(\zeta=1\); Overdamped system where \(\zeta>1\) (Rix and Valdes, n.d.).

2.1.1 Underdamped System

The displacement time continues for many cycles of motion which can be considered to be a harmonic function modulated by a decreasing exponential function (Rix and Valdes, n.d.).
Damped circular natural frequency is defined as

$$\omega_d = \omega_0 \sqrt{1 - \zeta}$$  \hspace{1cm} (2.18)

### 2.1.2 Critically Damped System

Since the damping ratio for the system $\zeta = 1$,

$$s = -\omega_0$$  \hspace{1cm} (2.19)

and the partial differential equation has repeated roots. As a result, the solution takes the form

$$x(t) = \left[x_0 + (\dot{x}_0 + \omega_0 x_0)\right] e^{-\omega_0 t}$$  \hspace{1cm} (2.20)

Since the exponential term in the equation is real, the system cannot oscillate.
2.1.3 Overdamped System

By following the procedure as in underdamped system,

\[ x(t) = e^{-\xi \omega_0 t} \left( A e^{\xi \sqrt{\omega^2 - 1} t} + B e^{-\xi \sqrt{\omega^2 - 1} t} \right) \]  \hspace{1cm} (2.21)

where \( A \) and \( B \) are the integration constants to be determined from initial conditions. Since there is no complex terms in the equation, the system is not able to make an oscillatory movement.

2.1.4 Logarithmic decrement

Damping can be categorized into active damping and passive damping. Active damping refers to energy dissipation from the system by external means while passive damping refers to energy dissipation within the structure by add-on damping devices or the internal damping. Logarithmic decrement method is used to determine the damping value in time domain. Logarithmic decrement is the natural logarithmic value of the ratio of two adjacent peak values of displacement in free decay vibration (Vibration Damping).

The rate of decrement of amplitude of a free damped vibration is represented by the logarithmic decrement (Griffin, 2001).

![Figure 2.3: Response of damped free vibration](https://example.com/image)

The rate of decrement of amplitude of a free damped vibration is represented by the logarithmic decrement (Griffin, 2001).

\[ x(t) = Ae^{-\xi \omega_0 t} \cos(\omega_d t - \phi) \]  \hspace{1cm} (2.22)

\[ X_1 = Ae^{-\xi \omega_0 t_1}, \quad X_2 = Ae^{-\xi \omega_0 t_2} = Ae^{-\xi \omega_0 (t_1 + T_d)} \]  \hspace{1cm} (2.23)
\[ \frac{X_1}{X_2} = \frac{A e^{-\zeta \omega_n t_d}}{A e^{-\zeta \omega_n (n+1) t_d}} = e^{\omega_n T_d} \]  

(2.24)

\[ \delta = \ln \frac{X_1}{X_2} = \zeta \omega_n T_d = \zeta \omega_n \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{2\pi \zeta}{\sqrt{1-\zeta^2}} \]  

(2.25)

if \( \zeta < 0.1 \) (lightly damped), \( \zeta = \frac{\delta}{2\pi} \). If \( \zeta > 0.1 \), \( \zeta = \frac{\delta}{2\pi} \sqrt{1+\left(\frac{\delta}{2\pi}\right)^2} \).

For measurement of logarithmic decrement from of a lightly damped system, take 2nd Amplitude after N cycles (Griffin, 2001).

\[ \frac{X_1}{X_{N+1}} = \frac{A e^{-\zeta \omega_n t_d}}{A e^{-\zeta \omega_n (n+1) t_d}} = e^{\omega_n N T_d} \]  

(2.26)

\[ \ln \frac{X_1}{X_{N+1}} = N(\zeta \omega_n T_d) = N \delta \]  

(2.27)

\[ \delta = \frac{1}{N} \ln \frac{X_1}{X_{N+1}} \]  

(2.28)

2.2 Interferometry

2.2.1 The Stationary Michelson Interferometer

The Michelson interferometer is as shown below:

![Figure 2.4: Schematic of the Michelson interferometer.](image)

Source: Blumberg, Thompson and Zaslavsky, 2006.
The interferometer compares the relative phase of two paths of the laser light, which are the path proceeding directly through the beam splitter, reflecting off the top mirror, and reflecting off the beam splitter to the detector, and the path initially reflecting off the beam splitter, then reflecting off the target, and passing through the beam splitter to hit the detector. The two paths cover different distances:

\[ d_1 = d_s + 2d_m + d_d \] (2.29)

for path 1

\[ d_2 = d_s + 2d_i + d_d \] (2.30)

for path 2. Therefore, the electric fields of the two beams will probably be out of phase upon arriving at the detector. Using the sinusoidal electromagnetic wave equation:

\[ \vec{E} = \vec{E}_0 \sin(kx - \omega t + \phi) \] (2.31)

the electric field strengths at the detector are

\[ \vec{E}_1 = \vec{E}_0 \sin(kd_1 - \omega t + \phi) \] (2.32)

\[ \vec{E}_2 = \vec{E}_0 \sin(kd_2 - \omega t + \phi) \] (2.33)

And the overall total electric field is

\[ \vec{E}_d = \vec{E}_0 \left( \sin(kd_1 - \omega t + \phi) + \sin(kd_2 - \omega t + \phi) \right) \] (2.34)

\[ \vec{E}_d = \vec{E}_0 \left( \sin(kd_1 + 2kd_m + kd_d - \omega t + \phi) + \sin(kd_1 + 2kd_i + kd_d - \omega t + \phi) \right) \] (2.35)

2.2.2 Changing Path Lengths in the Interferometer

The path length component \( d_i \) varies in time. Allowing \( d_m \) and \( \vec{E}_d(t) \) to be time-dependent and split the distances \( d_m(t) \) and \( d_i(t) \) into constant and variable components:

\[ d_m(t) = d_{m0} + \delta_m(t) \] (2.36)

\[ d_i(t) = d_{i0} + \delta_i(t) \]  \hspace{1cm} (2.37)

to get
\[ E_d = E_0 \left( \sin(kd_i + 2kd_m + kd_o - \omega t + \phi + 2k\delta_m(t)) + \sin(kd_i + 2kd_i + kd_o - \omega t + \phi + 2k\delta_i(t)) \right) \]  \hspace{1cm} (2.38)

Equation (2.38) includes terms representing oscillations on two different orders of magnitude. The oscillation of the laser, represented by \( \omega \), will be nearly \( 10^{15} \text{Hz} \) for the visible-light lasers that will be available in practice. Therefore \( \delta_m(t) \) and \( \delta_i(t) \) can be treated as approximately constant over a few oscillations of the laser radiation. Zero point of time and the quantities \( d_{m0} \) and \( d_{i0} \) can be defined such that \( kd_i + 2kd_m + kd_o - \omega t + \phi \) and \( kd_i + 2kd_i + kd_o - \omega t + \phi \) are integral multiples of \( 2\pi \). This reduces equation (2.38) to
\[ E_d(t) = E_0 \left( \sin(2k\delta_m(t)) + \sin(2k\delta_i(t)) \right) \]  \hspace{1cm} (2.39)

2.2.3 Optimizing Intensity Variation

The detector measures the electromagnetic intensity, given by
\[ I(t) \propto E_d^2(t) \]  \hspace{1cm} (2.40)
\[ I(t) \propto E_0^2 \left( \sin(2k\delta_m(t)) + \sin(2k\delta_i(t)) \right)^2 \]  \hspace{1cm} (2.41)

Differentiating \( I(t) \), for a small change in \( \delta_i(t) \) the intensity variation is
\[ \frac{\partial I}{\partial \delta_i} \propto kE_0^2 \left( \sin(2k\delta_m(t)) + \sin(2k\delta_i(t)) \right) \cos(2k\delta_i(t)) \]  \hspace{1cm} (2.42)

The interferometer is set up to maximize the magnitude of \( \frac{\partial I}{\partial \delta_i} \) and thus provide the greatest detectable signal, so
\[ \frac{\partial^2 I}{\partial \delta_m \partial \delta_i} \propto 8k^2 E_0^2 \cos(2k\delta_m(t)) \cos(2k\delta_i(t)) = 0 \]  \hspace{1cm} (2.43)
\[ \cos(2k\delta_m(t)) = 0 \text{ or } \cos(2k\delta_i(t)) = 0 \]  \hspace{1cm} (2.44)
REFERENCES


Binua, S., Mahadevan V.P. Pillaia & Chandrasekaranb N. 2006. Fibre optic displacement sensor for the measurement of amplitude and frequency of vibration. Department of Optoelectronics, University of Kerala.

Chow, H.W. & Cheung N. 2009. Low cost displacement sensor at sub-micron precision based on 3 x3 optical coupler for vibrating surface. Department of Electrical Engineering. The Hong Kong Polytechnic University.


p_LT6202_Aug02_Mag.pdf