FOURTH-ORDER EXPLICIT RUNGE-KUTTA AND
SECOND-ORDER EXPLICIT ADAM-BASHFORTH
METHODS FOR SOLVING
NONLINEAR DYNAMIC SYSTEM: FROUDE’S PENDULUM

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ABSTRAK

ABSTRACT

In this dissertation, the Fourth Order Explicit Runge Kutta method (ERK4) and Second Order Explicit Adam Bashforth method (EAB2) are implemented in solving nonlinear dynamic system such as the Froude pendulum. Comparisons between ERK4 method and EAB2 method will be made to clarify the effectiveness of these two methods to achieve accuracy. Different step sizes are also considered in this dissertation. Besides, a C-language based program will be implemented. From the obtained results, Fourth Order Explicit Runge Kutta Method (ERK4) is found to be more accurate than Second Order Explicit Adam Bashforth Method (EAB2).
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td>ii</td>
</tr>
<tr>
<td>AUTHENTICATION</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>CONTENT</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xiv</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong> <strong>INTRODUCTION</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 Introduction of Ordinary Differential Equation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Nonlinear Dynamic System</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Froude Pendulum</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Application of Froude Pendulum</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Runge-Kutta Method</td>
<td>10</td>
</tr>
<tr>
<td>1.6 Multistep Method</td>
<td>12</td>
</tr>
<tr>
<td>1.7 Aims And Objective</td>
<td>14</td>
</tr>
<tr>
<td>1.8 Scope of Research</td>
<td>14</td>
</tr>
<tr>
<td><strong>CHAPTER 2</strong> <strong>LITERATURE REVIEW</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 The Evolution of The Pendulum</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Previous Studies On Froude Pendulum</td>
<td>17</td>
</tr>
<tr>
<td>2.3 Previous Studies On Nonlinear Dynamic System</td>
<td>18</td>
</tr>
<tr>
<td>2.4 Previous Studies On Runge-Kutta Method</td>
<td>19</td>
</tr>
<tr>
<td>2.4.1 The Explicit Runge-Kutta Method</td>
<td>19</td>
</tr>
<tr>
<td>2.4.2 The Implicit Runge-Kutta Method</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Previous Studies On Multistep Method</td>
<td>21</td>
</tr>
<tr>
<td>2.5.1 The Explicit Adam-Bashforth Method</td>
<td>22</td>
</tr>
<tr>
<td>2.5.2 The Implicit Adam-Moulton Method</td>
<td>23</td>
</tr>
</tbody>
</table>
CHAPTER 3 METHODOLOGY

3.1 Introduction

3.2 Equation of Motion for A Free Froude Pendulum

3.3 Formulation of The Runge-Kutta Methods
   3.3.1 First-Order Runge-Kutta Method
   3.3.2 Second-Order Runge-Kutta Method
   3.3.3 Third-Order Runge-Kutta Method
   3.3.4 Fourth-Order Runge-Kutta Method
   3.3.5 High-Order Runge-Kutta Method

3.4 Formulation of The Adam-Bashforth Methods
   3.4.1 First-Order Adam-Bashforth Method
   3.4.2 Second-Order Adam-Bashforth Method
   3.4.3 High-Order Adam-Bashforth Method

3.5 Absolute Stability
   3.5.1 Stability Region for Fourth-Order Runge-Kutta Method
   3.5.2 Stability Region for Second-Order Adam-Bashforth Method

3.6 Error Analysis
   3.6.1 Relative Error

3.7 The Coefficient of Determinant, $R^2$

3.8 Discretization of The Froude Pendulum

3.9 Discretization of ERK4 Method
   3.9.1 Algorithm for ERK4 Method

3.10 Discretization of EAB2 Method
   3.10.1 Algorithm for EAB2 Method

CHAPTER 4 RESULTS

4.1 Overview

4.2 Numerical Results
   4.2.1 Comparisons of Errors Versus Number of Iterations at Different Step Sizes
CHAPTER 5       DISCUSSIONS AND CONCLUSION

5.1 Discussion
5.2 Summary And Conclusion
5.3 Suggestion for Further Work

REFERENCES
<table>
<thead>
<tr>
<th>Table No.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Second order explicit RK scheme in Butcher Tableau</td>
<td>34</td>
</tr>
<tr>
<td>3.2</td>
<td>Third order explicit RK scheme in Butcher Tableau</td>
<td>37</td>
</tr>
<tr>
<td>3.3</td>
<td>Fourth order explicit RK scheme in Butcher Tableau</td>
<td>43</td>
</tr>
<tr>
<td>3.4</td>
<td>Minimum s and number of RKM error coefficients for orders to 8</td>
<td>44</td>
</tr>
<tr>
<td>3.5</td>
<td>Coefficients of Adam Bashforth methods up to order 6</td>
<td>47</td>
</tr>
<tr>
<td>4.1</td>
<td>Comparisons of theta versus number of iterations at different step sizes with different equations and values of $R^2$</td>
<td>74</td>
</tr>
<tr>
<td>4.2</td>
<td>Comparisons of function evaluations, $k$ versus number of iterations at different step sizes with different equations and values of $R^2$</td>
<td>79</td>
</tr>
<tr>
<td>4.3</td>
<td>Comparisons of function evaluations, $f$ versus number of iterations at different step sizes with different equations and values of $R^2$</td>
<td>82</td>
</tr>
<tr>
<td>4.4</td>
<td>Comparisons of relative error between ERK4 and EAB2 methods at different step sizes with different equations and values of $R^2$</td>
<td>84</td>
</tr>
<tr>
<td>4.5</td>
<td>Comparisons of error versus number of iterations at different step sizes with different equations and values of $R^2$</td>
<td>90</td>
</tr>
<tr>
<td>4.6</td>
<td>Absolute stability condition for ERK4 method with $h=0.1$, $h=0.2$ and $h=0.3$</td>
<td>92</td>
</tr>
<tr>
<td>4.7</td>
<td>Absolute stability condition for EAB2 method with $h=0.1$ and $h=0.2$ and $h=0.3$</td>
<td>96</td>
</tr>
<tr>
<td>Figure No.</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1.1</td>
<td>The structure of hand water pump with pendulum</td>
<td>6</td>
</tr>
<tr>
<td>1.2</td>
<td>Side view of the pump and the position of the piston, lever and the pendulum during operation of the pump</td>
<td>6</td>
</tr>
<tr>
<td>1.3</td>
<td>The structure of pendulum clock</td>
<td>9</td>
</tr>
<tr>
<td>3.1</td>
<td>Elements of the methodology</td>
<td>25</td>
</tr>
<tr>
<td>3.2</td>
<td>Illustration of a Froude pendulum</td>
<td>26</td>
</tr>
<tr>
<td>3.3</td>
<td>Stability plot of the Runge-Kutta method</td>
<td>52</td>
</tr>
<tr>
<td>3.4</td>
<td>Stability regions for Adam-Bashforth method</td>
<td>56</td>
</tr>
<tr>
<td>3.5</td>
<td>Flowchart of C++ coding for ERK4 method</td>
<td>65</td>
</tr>
<tr>
<td>3.6</td>
<td>Flowchart of C++ coding for EAB2 method</td>
<td>69</td>
</tr>
<tr>
<td>4.1</td>
<td>Theta versus the number of iterations of ERK4 and EAB2 methods with ( h=0.1 ) and ( w[0]=1 )</td>
<td>71</td>
</tr>
<tr>
<td>4.2</td>
<td>Theta versus the number of iterations of ERK4 and EAB2 methods with ( h=0.2 ) and ( w[0]=1 )</td>
<td>72</td>
</tr>
<tr>
<td>4.3</td>
<td>Theta versus the number of iterations of ERK4 and EAB2 methods with ( h=0.3 ) and ( w[0]=1 )</td>
<td>72</td>
</tr>
<tr>
<td>4.4</td>
<td>Function evaluations, ( k ) versus the number of iterations of ERK4 method with ( h=0.1 ) and ( w[0]=1 )</td>
<td>75</td>
</tr>
<tr>
<td>4.5</td>
<td>Function evaluations, ( k ) versus the number of iterations of ERK4 method with ( h=0.2 ) and ( w[0]=1 )</td>
<td>76</td>
</tr>
<tr>
<td>4.6</td>
<td>Function evaluations, ( k ) versus the number of iterations of ERK4 method with ( h=0.3 ) and ( w[0]=1 )</td>
<td>76</td>
</tr>
<tr>
<td>4.7</td>
<td>Function evaluations, ( f ) versus the number of iterations of EAB2 method with ( h=0.1 ) and ( w[0]=1 )</td>
<td>80</td>
</tr>
<tr>
<td>4.8</td>
<td>Function Evaluation, ( f ) versus the number of iterations of EAB2 method with ( h=0.2 ) and ( w[0]=1 )</td>
<td>80</td>
</tr>
<tr>
<td>4.9</td>
<td>Function Evaluation, ( f ) versus the number of iterations of EAB2 method with ( h=0.3 ) and ( w[0]=1 )</td>
<td>81</td>
</tr>
</tbody>
</table>
4.10 Relative error versus the different step sizes of each ERK4 and EAB2 methods

4.11 Error versus the number of iterations of ERK4 and EAB2 methods with \( h=0.1 \) and \( w[0]=1 \)

4.12 Error versus the number of iterations of ERK4 and EAB2 methods with \( h=0.2 \) and \( w[0]=1 \)

4.13 Error versus the number of iterations of ERK4 and EAB2 methods with \( h=0.3 \) and \( w[0]=1 \)

4.14 Absolute Stability condition for ERK4 method with \( h=0.1 \)

4.15 Absolute Stability condition for ERK4 method with \( h=0.2 \)

4.16 Absolute Stability condition for ERK4 method with \( h=0.3 \)

4.17 Absolute Stability condition for EAB2 method with \( h=0.1 \)

4.18 Absolute Stability condition for EAB2 method with \( h=0.2 \)

4.19 Absolute Stability condition for EAB2 method with \( h=0.3 \)
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ODEs</td>
<td>Ordinary Differential Equations</td>
</tr>
<tr>
<td>RKM</td>
<td>Runge Kutta -Method</td>
</tr>
<tr>
<td>RK4</td>
<td>Runge Kutta-Method of Fourth Order</td>
</tr>
<tr>
<td>ERK</td>
<td>Explicit Runge-Kutta</td>
</tr>
<tr>
<td>EAB</td>
<td>Explicit Adam-Bashforth</td>
</tr>
<tr>
<td>ABM</td>
<td>Adam-Bashforth Method</td>
</tr>
<tr>
<td>LMM</td>
<td>Linear Multi Method</td>
</tr>
<tr>
<td>ERK4</td>
<td>Explicit-Runge Kutta Method of Fourth Order</td>
</tr>
<tr>
<td>EAB2</td>
<td>Explicit Adam-Bashforth Method of Second Order</td>
</tr>
<tr>
<td>SSR</td>
<td>Regression</td>
</tr>
<tr>
<td>SSE</td>
<td>Error variation</td>
</tr>
<tr>
<td>SStotal</td>
<td>Total variation</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

\( s \)  
order of Runge-Kutta method.

\( P(s) \)  
evaluations of \( f \) for each timestep

\( M(\Omega - \theta) \)  
frictional torque that related to slipping velocity \( \theta \)

\( m \)  
mass of pendulum

\( I \)  
total moment of inertia

\( l \)  
distance from the axis of rotation to the center of gravity

\( c \)  
viscous coefficient due to the resistance of the air.

\( \Omega \)  
angular velocity of the rotating shaft.

\( w \)  
weighting coefficients

\( v \)  
order of accuracy of the RK method

\( c_i \)  
quadrature nodes

\( b_i \)  
quadrature weights

\( a_{ij} \)  
Runge-Kutta coefficients

\( s \)  
number of stages

\( \xi_j \)  
root of the characteristic polynomial

\( P(r) \)  
Stability polynomial of the RK process

\( \frac{d\theta}{d\tau} \)  
First order derivatives towards a function \( \theta \) of \( \tau \)

\( \frac{dw}{dy} \)  
First order derivatives towards a function \( w \) of \( y \)

\( \frac{dy}{dx} \)  
First order derivatives towards a function \( y \) of \( x \)

\( \frac{d^2y}{dx^2} \)  
Second order derivatives towards a function \( y \) of \( x \)

\( \frac{d^2\theta}{d\tau^2} \)  
Second order derivatives towards a function \( \theta \) of \( \tau \)

\( E_{\text{max}} \)  
Maximum possible absolute error

\( R^2 \)  
The coefficient of the determination

\( \lambda \)  
Eigenvalues
\( h \)  
Step size

\( r \)  
Absolute stability condition
CHAPTER 1

INTRODUCTION

1.1 Introduction of Ordinary Differential Equation

Differential equation is a type of equation which is expressed in terms of unknown function of several variables that relates the values of the function itself and their derivative. Differential equations commonly have been applied in engineering, physics, economics, and chemistry (Dormand, 1996).

For example, the mathematical models implement the fundamental scientific laws of physics which include principle of conservation of linear momentum, principle of conservation of mass, and principal of conservation of energy in the term of differential equation (Reddy, 2004).

If a solution for a given differential equation is not readily available, the numerical methods are considered to determine the exact form. By using numerical method, this method can compute a solution of accuracy to the differential equation over of period of time as well as solving many practical problems of science and
engineering. A well-known example is simple pendulum which based on the principle of conservation of linear momentum (e.g. Newton’s second law of motion) to determine the motion of pendulum (Dormand, 1996; Reddy, 2004)

There are many numerical methods for solving ordinary differential equations (ODEs). These methods are usually categorized as one-step and multistep methods. One of the explicit one-step methods is Runge-Kutta methods. On the other hand, the multistep methods include implicit Adam-Moulton, explicit Adam-Bashforth and Predictor-corrector methods. The approach that used by one-step method is totally different as compared to multistep methods. One-step methods use the solution at a single initial point to get an approximation to the solution of next point. Multistep methods use the sequence of previous solution and derivative values (Dormand, 1996).

There are five types of differential equation; ordinary difference equation, partial difference equation, delay differential equation, stochastic differential equation, and differential algebraic equation. Each of these categories lead to linear and nonlinear system. A differential equation is considered to be linear when the dependent variable and its derivatives occur only to the power one and there are no functions for dependent variable. Otherwise the particular differential equation is considered as nonlinear equation (Wikipedia, 2007a; Dormand, 1996).

Regarding the history of differential equation, Leonhard Euler who also invented Euler’s method introduced the theory of differential equations in the year 1768. In the year 1824, Augustin Louis Cauchy used implicit Euler method to prove...
convergence of Euler method. John Couch Adams was the first person who designed the multistep method in 1855. After 40 years in 1895, Carle Runge introduced the first Runge-Kutta method. In 1905, Martin Kutta developed the fourth order Runge-Kutta method. During the year of 1910, Lewis Fry Richardson introduced his extrapolation method (Wikipedia, 2007b).

1.2 Nonlinear Dynamic System

Dynamic system is the behavior of one or more particles that modelled by differential equation with initial condition. The behaviour of dynamic system such as velocity, position and acceleration. The equation of motion for Froude pendulum is shown as follow: (De Jong, 1991).

\[
\frac{d}{dt} \left( \frac{d^2 \theta}{dt^2} \right) + (c + M'(\Omega)) \frac{d\theta}{dt} + mgl \sin \theta + \frac{1}{6} M''(\Omega) \left( \frac{d\theta}{dt} \right)^3 = M(\Omega)
\]  

(1.1)

with initial condition: \( \theta(0) = 0 \).

From the Froude pendulum in equation (1.1), it can be found that it is nonlinear dynamic system because the differential equations involve non-linear terms like \( \sin \theta \) and \( \left( \frac{d\theta}{dt} \right)^3 \). The restoring force of the pendulum that is \( mgl \sin \theta \) where \( \theta \) is the angle of displacement. If the angle of displacement is smaller than \( 15^\circ \), the \( \sin \theta \) is just approximated with simply \( \theta \). On the contrary, approximation of \( \sin \theta \) is inaccurate and it is considered as nonlinear system if the angle is larger than \( 15^\circ \) (Davidson, 2005).
Nonlinear systems seem to provide real natural systems unlike linear systems that make simple equations and behaviour of simple system such as simple pendulum in phase space. Mostly, there are some factors often vary in nonlinear system such as friction forces, damping elements, resistive elements in circuits. It seems that the solution of nonlinear system is very messy if comparing with linear system. The analytic solution to these systems are difficult to be obtained due to the exponential increase error in measurement when the system progresses. As a result, numerical method can only be applied to solve nonlinear system (Davidson, 2005).

1.3 Froude Pendulum

Froude’s pendulum is a classical dynamic system that generates self-oscillations in a mechanical system with friction. The shaft of the Froude pendulum is connected to an engine that rotates at an angular velocity. It fixed to bearing pivot which swings along the rotating shaft (Dai & Singh, 1998).

Self-excited oscillations are produced due to the friction force between a rotating shaft and the bearing pivot of a Froude pendulum. The increasing amplitude of pendulum depends on damping force and frequency of rotation. This type of pendulum is different from the simple pendulum because of its complex three dimensional motion (Dai & Singh, 1998; Djidjeli et al., 1996).
For simple pendulum, the motion occurred in a two dimension plane such that it does not swing into and out of the page. The actual form of a simple pendulum consists of a long bar suspended at the end of a string that support a small, massive bob. The simple pendulum body suspends from a fixed point so that it can swing under influence of gravity and under its own momentum (Wikipedia, 2008).

1.4 Application of Froude pendulum

(i) Hand Water Pump With Pendulum.

Hand water pump with pendulum is a realization of a new and technically original solution for pumping water. The pumping works can be accomplished with less invested energy in the form of pushing of the pendulum. The pendulum generates about 12 times more output energy than the manual input energy required to keep the pendulum swinging. It seems that the pumping work can become easier, long-lasting and effortless if using hand water pump. Hence a user can pump out a 1200 liters of water per hour, without any fatigue and continue with the pumping (Milkovic, 2005).

From Figure 1.1 and 1.2, hand water pump with a pendulum for pumping water consists of a cylinder (1) with a piston (2), lever system (3), a seesaw (4), a pendulum (5), a reservoir (6) and output water pipe (7) (Milkovic, 2005).
Figure 1.1  The Structure Of Hand Water Pump With Pendulum (Milkovic, 2005).

Figure 1.2  Side View Of The Pump And The Position Of The Piston, Lever And The Pendulum During Operation Of The Pump (Milkovic, 2005).
In order to move the piston and lever vertical in the up-down direction, it can give the pendulum which is attached to the end of the lever system a push so that much less effort is needed to pump out water. The action of the hands is required to maintain the amplitude of the oscillation for a continuous intensity of water flow. The pump works well with all sizes of the pendulum, but the best performance is achieved with an amplitude of about 90°. Besides, the see-saw and the piston also oscillating together with the pendulum, the side with the piston should be heavier than the side with the pendulum. It is about 50% heavier than the centrifugal force of the pendulum when its swinging amplitude is 90° (Milkovic, 2005).

To get the water coming out of the output pipe, the pendulum needs to be out of balance and should be occasionally pushed to maintain the amplitude of water. The power of gravitational potential is also used as driving power. With the gravitational potential, the piston will rise to the highest point and the pendulum passes through the bottom vertical position during the swinging. In these moments, the pendulum will overweigh on the see-saw position and in this phase the piston will push the continuous stream of water out of the output pipe (Milkovic, 2005).
The advantages of this new technically hand water pump compared to present hand pump are (Ecosustainable, 2006).

i) needs the minimum of the effort, because it is only necessary to swing the pendulum and maintain the oscillation for several hours, without any fatigue.

ii) Both elderly and children can use it because maintaining the oscillation of the pendulum is a easy work and it does not request any special training. So everyone can do it.

iii) The pendulum can be a children’s swing, so that useful work can also be done through their playing.

ii) Pendulum Clock.

A pendulum clock is a clock that uses a pendulum and a swinging weight to calculate the accuracy of time period. From its invention in 1656 by Christiaan Huygens until the 1930s, the pendulum clock can be considered as the world’s most accurate timekeeper. Pendulum clocks must be stationary to operate; any motion or accelerations will affect the motion of the pendulum, causing inaccuracies, so other mechanisms must be used in portable timekeepers. They are now kept mostly for their decorative and antique value (Ventures, 2004).
REFERENCES


