ADM IN SOLVING FOURTH ORDER PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

WONG HOCK MING

THIS DISSERTATION IS SUBMITTED AS PART OF THE PRE-REQUIREMENT TO BE CONFERRED THE DEGREE IN BACHELOR OF SCIENCE HONOURS

MATHEMATICS WITH COMPUTER GRAPHICS PROGRAMME
SCHOOL OF SCIENCE AND TECHNOLOGY
UNIVERSITI MALAYSIA SABAH

APRIL 2007
BORANG PENGESAHAN STATUS TESIS

JUDUL: ADM in solving fourth order parabolic Partial Differential Equations.

Ijazah: Sarjana Muda, Dengan Kekajiann matematik Dengan komputer Cina

SESI PENGAJIAN: 04 → 07

Saya WONG HOCK MING

(HURUF BESAR)

mengaku membenarkan tesis (LPS/Sarjana/Doktor Falsafah)* ini disimpan di Perpustakaan Universiti Malaysia Sabah dengan syarat-syarat kegunaan seperti berikut:

1. Tesis adalah hakmilik Universiti Malaysia Sabah.
2. Perpustakaan Universiti Malaysia Sabah dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan tesis ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. **Sila tandakan ( / )
   - [ ] SULIT
   - [X] TERHAD
   - [ ] TIDAK TERHAD

(Mengandungi maklumat yang berdasarkan keselamatan atau kepentingan Malaysia seperti yang termaktub di dalam AKTA:RAHSIA RASMI 1972)

(Mengandungi maklumat TERHAD yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)

(TANDATANGAN PENULIS)

Alamat Tutan: No 2B, Lorong Kiew Nang 6,

Tanggal: 23 April 2007

(TANDATANGAN PUSTAKAWAN)

Nama Penyelidik:

Miss Suvelawati

Tanggal: 23/4/07

CATATAN:
- * Potong yang tidak berkenaan.
- ** Jika tesis ini SULIT atau TERHAD, sila lampirkan surat daripada pihak berkuasa/organisasi berkenaan dengan menyatakan sekali sebuh dan tempoh tesis ini perlu dikeluarkan sebagai SULIT dan TERHAD.
- @ Tesis dimaksudkan sebagai tesis bagi ijazah Doktor Falsafah dan Sarjana secara penyelidikan, atau disertasi bagi pengajian secara kerja kursus dan penyelidikan, atau Laporan Projek Sarjana Muda (LPSM).
DECLARATION

I, hereby declare that this is my original work except for quotation that I have clearly noticeable for sources.

APRIL 2007

WONG HOCK MING
HS 2004 - 4559
VERIFIED BY

1. SUPERVISOR
   (MS. SUZELAWATI ZENIAN)

2. EXAMINER
   (ASSOC. PROF. DR. HO CHONG MUN)

3. DEAN
   (SUPT/KS. ASSOC. PROF. DR. SHARIFF. A.K. OMANG)
ACKNOWLEDGEMENT

I would like to take this opportunity to thank to my supervisor, Miss Suzelawati Zenian, for her guidance, advices, support and a lot of help during processing the project. All her effect and sacrifice in time are deeply appreciated and would not be forgotten.

I would like to thank to all lecturers of Mathematics programme for giving me precious knowledge during my years in UMS. Every lecturer has contributed in giving me supportive advice, moral support, and assistance. Without them, I am sure I could not accomplish this project.

Not forgetting my appreciation towards the staff of School of Science and Technology in smoothly arranging the process for final year project and also their assistance.

Lastly, I would like to express my deepest gratitude towards the full support, love and encouragement by my parents, family member, and fellow friends along. Thanks to all.
ABSTRAK

ABSTRACT

The standard Adomian Decomposition Method and modified Adomian Decomposition Method are the methods that been proposed recently. Thus, both methods are implemented in fourth order partial differential problems. The modified ADM has less calculation than standard ADM. The excellent performance in obtaining the exact solution by the Adomian Method. It also contributes to the rapid convergence and accuracy in solving partial differential equation. The ADM is very useful for many application in science.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECLARATION</td>
<td>ii</td>
</tr>
<tr>
<td>VERIFICATION</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF CONTENT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>vii</td>
</tr>
</tbody>
</table>

### CHAPTER 1 INTRODUCTION

1.1 History Of George Adomian                  | 1    |
1.2 Numerical Analysis                         | 2    |
1.3 Adomian Decomposition Method               | 3    |
    1.3.1 The Standard Adomian Decomposition Method | 4    |
    1.3.2 The Modified of Adomian Decomposition Method | 6    |
    1.3.3 The Two-Step of Adomian Decomposition Method | 8    |
1.4 Objective Of The Research                  | 9    |
1.5 Scope Of The Research                      | 9    |

### CHAPTER 2 LITERATURE REVIEW

2.1 Application Of The Standard Adomian Decomposition Method | 11   |
2.2 Application Of The Modified Of Adomian Decomposition Method | 14   |
2.3 Application Of The Two Step Of Adomian Decomposition Method | 16   |

### CHAPTER 3 METHODOLOGY

3.1 Introduction                               | 17   |
3.2 Methodology of Standard Adomian Decomposition Method | 18   |
    3.2.1 Application Standard Adomian Decomposition Method in Linear Equation | 19   |
    3.2.2 Standard Adomian Decomposition Method in Non-Linear Equation | 21   |
3.3 Methodology and Application of Modification of Adomian Decomposition Method | 23   |
CHAPTER 4 RESULT

4.1 Introduction 25
4.2 Fourth Order Homogenous Partial Differential Equation Problem 25
   4.2.1 Solution By Standard ADM 26
   4.2.2 By Modified ADM 33
4.3 Fourth Order Nonhomogenous Partial Differential Equation Problem 37
   4.3.1 By Standard ADM 37
   4.3.2 By Modified ADM 41

CHAPTER 5 DISCUSSION AND CONCLUSION

5.1 Introduction 44
5.2 Discussion 45
   5.2.1 Standard Adomian Decomposition Method 45
   5.2.2 Modified ADM 47
5.3 Suggestion 49
5.4 Conclusion 49

REFERENCES 51
<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Element of the Methodology</td>
<td>18</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------</td>
<td></td>
</tr>
<tr>
<td>FSLE</td>
<td>Fuzzy system of linear equation</td>
<td></td>
</tr>
<tr>
<td>ADM</td>
<td>Adomian Decomposition Method</td>
<td></td>
</tr>
<tr>
<td>TSADM</td>
<td>Two step of Adomian Decomposition Method</td>
<td></td>
</tr>
<tr>
<td>UGA</td>
<td>University of Georgia</td>
<td></td>
</tr>
</tbody>
</table>
1.1 History Of George Adomian

George Adomian, who is an Armenian American Mathematician, proposed the Adomian Decomposition Method (ADM) in 1980s to solve the problems of many fields in applied science. (Abbasbandy, 2005) He born in 1922 and died in 1996.

Adomian contributed to many fields especially in solving mathematical problems. He also modified the method to improve the accuracy and efficiency of the solutions. There are three type of Adomian Decomposition Method, which are standard Adomian Decomposition Method, Modified Adomian Decomposition Method, and Two-step of Adomian Decomposition Method (TSADM).

He is also the member of faculty of University of Georgia (UGA) since 1966 until 1989. Besides, he is also an aerospace engineer.
1.2 Numerical Analysis

Numerical analysis is the study and analysis on algorithms in obtaining the solutions for mathematical problems. Many results and methods from many areas of mathematics, particularly those of calculus and linear algebra, are applied by the numerical analysis. There are algorithms where always linked to computer program and very useful for programmer to apply in solving mathematical problems.

This algorithm is always implemented in the program or codes, which are needed to communicate between algorithm and computer. Scientific computation is the way to solve mathematical problem and numerical problem on computer. There are several computer languages to write program by using the numerical methods. This high-level languages allows users to manipulate the need in mathematical structures and implement numerical analysis procedures for solving mathematical problems.

In this term, there are constructive proof and nonconstructive proof in mathematics. Nonconstructive methods are always used in their uniqueness and existence theorems might be established by logic argument until a contradiction is existing. To lead the constructive proof to a success may face plenty of difficulties. Therefore, an analytical solution to a given problem may be several steps away from a numerical solution.

There are difference between an existence theorem and a numerical solution of the problem. For example, matrix equation \( Ax = b \), it has unique solution whenever
matrix $A$ is nonsingular. When we met with a large linear system containing empirical data, we have to compute an approximate numerical solution. In this case, numerical methods are necessary to be used in computing the approximate answers.

Numerical methods are commonly used as characteristics method needed large size of computation works and usually the round-off error causes the loss of accuracy. Adomian Decomposition Method has been applied to solve many functional equations and systems of functional equations. This method can transform the equations into a recursive relation. The Adomian has proven to be very effective and results in considerable saving in computation time.

Generally, each topic with a basic mathematical problem arises frequently in practical applications. Then a certain amount of analysis will be presented to arrive at an algorithm for solving problems.

Finally, additional algorithm may be shown to help in understanding its behavior, such as its convergence or resistance to corruption by round-off error. Such analysis may take the form of forward or backward analysis.

1.3 Adomian Decomposition Method

There are three type of Adomian Decomposition Method, which are standard Adomian Decomposition Method, Modified of Adomian Decomposition Method and also Two-step of Adomian Decomposition Method.
1.3.1 The Standard Adomian Decomposition Method

The Adomian Decomposition Method, first proposed by Adomian to solve the problems of linear and nonlinear models, physical, biological and chemical reactions, parabolic equation, and many other equations which can be applied in science.

In this scheme, the solution is performed in the form of a convergent power series with easily computable components. It is particularly concerned with the Adomian decomposition method and the results obtained are compared to those obtained by other conventional method. The numerical results demonstrate that the new method is relatively accurate and easily implemented. (Dogan Kaya, 2004) This method is relatively new and useful for obtaining both analytic and numerical approximations of linear or nonlinear differential equations and it is suitable for writing computer codes.

Numerical comparison of particular solutions in the decomposition method for linear and nonlinear problems indicates that there is a good communication between accuracy and efficiency of the numerical solutions and particular exact solutions. Analytical methods commonly used to solve many equations, which are very restricted and numerical technique involving discretion of the variables on the other hand gives rise to rounding off error.

For linear models, the Adomian Decomposition Method has been applied to obtain formal solutions. The efficiency and effectiveness of Adomian Decomposition Method are well-known for many kinds of equations. The method has a useful
attraction in that it provides the solution as an infinite series which each term can be easily determined. The series is quickly convergent towards an accurate solution.

For nonlinear models, the method has shown reliable results in supplying analytical approximation that converges very rapidly. For example, this method is to decompose the nonlinear term in the equations into a peculiar series of Adomian polynomials, which formally introduced to generate Adomian polynomials for all terms of nonlinearity. In this case, there is an alternative technique that develop to calculate Adomian polynomials in a practical way without any need for the formulas introduced by Adomian. It needs quite a huge size calculation to calculate Adomian polynomials for nonlinear terms in the differential equations (Biazar et al., 2005).

The nonlinear equation are solved easily and elegantly without transforming the equations by using Adomian decomposition method. The technique has many advantages over the classical techniques, it is to avoid linearization and perturbation. By this, it is use to find explicit solutions of a given nonlinear equations.

In general, there is no method which yields an exact solution for nonlinear fractional differential equations. Only the approximate solutions can be derived by using linearization or perturbation method. The advantage of the decomposition method is that it does not change the problem into a convenient one for the use of linear theory. However, the behavior of the numerical solution in limit by the time variable on some fixed mesh. The condition of the preservation of the nonnegative and convexity of the initial function. Then the condition of the conservation of the monotonic in the space variable of the initial function to the numerical solution.
In this section, Adomian Decomposition Method (ADM) is applied to determine the solutions of linear and nonlinear equations. ADM leads to approximate and exact solution of nonlinear equations easily without transforming to linear form or equations.

As a numerical tool, the method provides the numerical solution without any discretization on the given equation and therefore it is not affected by computation round-off errors. Straightforward computer codes in a symbolic programming language such as Maple or Mathlab can be written for reducing considerably the computing memory and time.

1.3.2 The Modified of Adomian Decomposition Method

Since 1980s, Adomian has developed a numerical technique for solving functional equations. He gives the solution as an infinite series usually converging to an accurate solution. Concrete applications to different functional equation are given by Adomian and his collaborators, Abbaoui and Cherruault, applied Adomian decomposition method in proving the convergence of the method to solve the equation \( f(x) = 0 \), where \( f(x) \) is a nonlinear function, and proved the convergence of the series solution. (Babolian et al, 2002)

A hypothesis on the convergence which is suitable and reasonable, is proposed and proved to accelerate the convergence. The modified form of the Adomian decomposition method will be implemented to construct such solution. This newly improvement technique introduces a promising tool and powerful improvement for
The modification presents a slight change in the definition of the component \( U_0 \) of the standard Adomian decomposition method. The idea can be extended to solve many kinds of equations.

1.3.3 Two-Step Of Adomian Decomposition Method

Since Adomian decomposition method is introduced, many problems of equations has been solved. Then the modified of Adomian decomposition method has a slight different from standard Adomian decomposition, but it perform a highly efficiency in computations.

In two-step of Adomian Decomposition Method, it makes further progress of the achievement than standard and modified of Adomian Decomposition Method. This method need only one iteration to solve the problems. In this term, Two-step Adomian Decomposition Method (TSADM) shows the less calculations than the previous methods. This proved the efficiency and effectiveness two-step Adomian Decomposition Method.

In addition, it is also slightly change from standard Adomian Decomposition Method but it may obtain exact solution for nonlinear equation without the need of Adomian polynomial. This method also shows the efficiency in computations of systems of inhomogeneous differential and integral, hyperbolic partial differential equations and singular initial value problems.
As a conclusion, two-step Adomian Decomposition Method has advantage over the standard Adomian Decomposition Method and can lead to a promising approach for many applications in science.

1.4 Objectives Of The Research

1. To determine and study the methods in solving fourth order parabolic partial differential equation

2. To obtain the solutions of fourth order problems by using the standard Adomian decomposition method and modified Adomian decomposition method.

3. The approximate analytical solutions can be demonstrated by the methods and the comparison can be made between the methods.

4. Then, the fourth order equations problems can also be tested whether can be solved either by standard Adomian Decomposition Method or Modified Adomian Decomposition Method.

1.5 Scope Of The Research

The research focuses on:

1. The applications and theory of the Adomian decomposition method especially in solving the problems of the fourth order parabolic partial differential equations
2. The standard Adomian decomposition method and modified of Adomian decomposition method are considered in the research.

3. The linear fourth order parabolic partial differential equations which consists of homogenous and non homogenous problems.
2.1 Application Of The Standard Adomian Decomposition Method

Dogan Kaya & Salah (2004) has conducted a research based on Adomian Decomposition Method to approximate the system of nonlinear equations are solved using the decomposition methods. As a result, the convergence of the algorithm is rapid. The equation is getting very efficient numerical solutions if more term are added into the decomposition series.

Wazwaz (2000) present an efficient numerical algorithm for approximate solutions of higher-order boundary value problems with two-point boundary condition. He motivated by desire to obtain numerical solutions to higher-order boundary value problems with a better accuracy level. The Adomian Decomposition Method was applied for obtaining approximate solutions to a wide class of differential and integral equations. In fact, it provides the solution in a rapidly convergent series with components that are elegantly computed and change accelerates the convergence of
the solution by reduces the volume of computational work. The main advantage of the method is that it can be used directly without using restrictive assumptions. He proved that the modified technique provides a qualitative improvement over standard Adomian Decomposition Method although it introduces a slight change in the formulation of Adomian recursive relation.

However, Wazwaz (1994) proposed the fourth order of parabolic partial differential equation, that governs the behavior of a vibrating beam, is solved by using the Adomian Decomposition Method. The solution is derived in the form of a power series with easily computable components. Comparing this methodology with some known techniques shows that the present approach is highly accurate.

Biazar & Ebrahimi (2005) applied Adomian decomposition method to solve hyperbolic partial differential equations. The result was compared to the characteristic method and the approximation can be obtained to any desired number of term. The small size in comparison with the computations in comparison with the computational size required in characteristics method and the rapid convergence show the improvement in solving hyperbolic equations.

Pamuk (2005) is saying about the exact solutions of a linear heat equation, and a nonlinear equation that usually arises in mathematical biology is obtained by using Adomian Decomposition Method. In addition, numerical comparison of particular solutions in the decomposition method for linear and nonlinear problems indicates that there is a good agreement between the numerical solutions and particular exact solutions in terms of accuracy and efficiency. In the research, the conclusion was that
Adomian Decomposition Method provides an accurate numerical solution for nonlinear problems in comparison with other methods. This method avoids linearization and physically unrealistic assumptions.

Luo. et al. (2006) discussed that the heat and wave equations are solved by partial solution technique. It shown that the fundamental equation of the method is well defined only for certain types of boundary conditions. In this scheme, if the boundary conditions are “well-defined”, the solution can be obtained by usual decomposition method. It presented a further insight into partial solutions in the decomposition method, and the resolution of such cases.

Bildik & Bayramoglu (2005) used Adomian Decomposition Method to investigate nonlinear, two dimensional wave equations. The research proved that the decomposition scheme obtain efficient results much closer to the actual solution. Clearly, the method can be applied to other complicated any other nonlinear problems.

Kamel Al-Khaled et al. (2004) applied Adomian Decomposition Method in solving the linear heat equation, which was based on a suitable differential operator. Comparison among this method and other conventional methods showed that this method is relatively accurate and easily implemented.

Babolian & Davari (2004) verify the use of numerical integration in Adomian Decomposition Method to evaluate the term of the series \( u = \sum u_i \), where this series is unable to solve by analytical methods.
Babolian & Biazar (2002) compare standard Adomian Decomposition Method with modified of Adomian decomposition method to solve the nonlinear equations. They conclude that modified of Adomian Decomposition Method can be extended to many nonlinear equations by deriving a sequence of approximations to the solution.

Ray & Bera (2005) apply Adomian Decomposition Method to solve the nonlinear fractional differential equation. They obtain the solution which has been numerically evaluated into the form of table. Comparison has been made with those obtained by truncated series method. As the outcome, they find that the approximate solution is more accurate by computing more term in the decomposition method.

Abbasbandy (2005) uses modified of Adomian Decomposition Method to create numerical algorithms in solving system of two nonlinear equations based on Newton’s method. The efficient modified of Adomian Decomposition Methods is proved to be applied in a system with more than two equations.

Jin & Liu (2005) proposed the new modification of Adomian Decomposition Method to solve the various kind of evolution equation. They evaluate the solutions of the problems and improve the convergence of the series solutions. The modified of Adomian Decomposition Method change slightly from the standard Adomian decomposition method but can be extended to solve many other kinds of equations.

Wazwaz (2000) find the approximate solutions for the higher order boundary value problems with two-point boundary conditions by using modified of Adomian Decomposition Method. This algorithm shows the convergence series is rapidly


