LINEAR ALGEBRA

• Ho Chong Mun • Toh Kheam Hoo • Amran Ahmed •

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Ho Chong Mun
Toh Kheam Hoo
Amran Ahmed

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Foreword

Although many textbooks can be found on any particular subject, it is difficult, if not impossible, to find one that exactly fulfills the requirements of a particular course or a particular set of students. We, the authors of this book, who are involved in the teaching of Mathematics at the School of Science & Technology (SST) in Universiti Malaysia Sabah (UMS), therefore felt that it would be beneficial for our students if we were to start a series of texts written expressly for their courses. The current book is the first result of this collaboration.

Associate Professor Dr. Ho Chong Mun has been involved in the teaching of Mathematics in Universiti Malaysia Sabah since 1996; Dr. Toh Kheam Hoo, a former Associate Professor in the Department of Mathematics (now Institute of Mathematical Sciences), Universiti Malaya, has been a 'flying' lecturer for various courses at the School of Science & Technology, Universiti Malaysia Sabah since 1996 and Associate Professor Dr. Amran Ahmed, the Deputy Vice Cancellor in Universiti Malaysia Sabah(Student Affairs and Alumni), has also been involved in Mathematical teaching since 1987, first in Universiti Kebangsaan Malaysia, then in Universiti Sains Malaysia and now in Universiti Malaysia Sabah.

This book on Linear Algebra is meant specifically for the course SM2043/ SH2143 for Universiti Malaysia Sabah students. As such, it covers the syllabus well and can be used as a classroom text. Be that as it may, a lecturer for the course may still use his discretion to modify certain parts of the course without difficulty. For example, if he feels that his current class may not need or are unable to assimilate proofs for all the theorems, he can leave them out without harming the continuity of his teaching; and of course he can look for more examples from other sources.

Although the book is written for a specific course, the material is essentially the same for any standard first course in Linear Algebra, and so we hope that other institutions may find it useful as well. It is written in English in response to the call of the Government to encourage the use of this language in the sciences.

The present version of this book is the first edition, and as such, there must be many faults with it. We therefore welcome feedback from students and teachers. Such information will help us, not only to improve the next edition, but also to have better guidelines for books on other courses yet to be written.

Ho Chong Mun
Toh Kheam Hoo
Amran Ahmed