A SUBCLASS OF CONVEX FUNCTIONS WITH RESPECT TO SYMMETRIC CONJUGATE POINTS

JOYCE LIM CHUEN SHIN

THIS DISSERTATION IS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH HONOURS

MATHEMATICS WITH COMPUTER GRAPHICS PROGRAM
SCHOOL OF SCIENCE AND TECHNOLOGY
UNIVERSITI MALAYSIA SABAH

2010

PUMS99:1

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JUDUL: A SUBCLASS OF CONVEX FUNCTIONS WITH RESPECT TO SYMMETRIC CONJUGATE POINTS

IJAZAH: SARJANA MUDA SAINS DENGAN KEPUJIAN (MATEMATIK DENGAN KOMPUTER GRAFIK)

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JOYCE LIM CHUEN SHIN

(BS07110092)

3 May 2010

CERTIFIED BY

Signature

1. **SUPERVISOR**
   (DR. AINI JANTENG)

2. **EXAMINER**
   (MISS KHADIZAH GHAZALI)

3. **DEAN**
   (PROF. DR. MOHD. HARUN ABDULLAH)

ACKNOWLEDGEMENT

This research project would not have been possible without the support and help from many people. First and foremost, I would like to express my deepest gratitude to my dearest supervisor, Dr. Aini Janteng for her helpful and valuable guidance, assistance, encouragement, advice and support throughout this final year project. I cannot fully express my appreciation for her generosity, enthusiasm and precious guidance.

Also I would like to express my greatest appreciation to my examiner, Miss Khadizah Ghazali. This study would not have been a success without her advice and comment.

Many thanks to all my fellow coursemates whose final year project are also under the supervision of Dr. Aini Janteng for sharing valuable information and knowledge. They have all been very helpful.

Last but not least, I would also like to thank my family members and friends for their understanding, everlasting mentally support, assistance and encouragement all the way throughout my studies.

JOYCE LIM CHUEN SHIN
BS07110092
3 May 2010

ABSTRACT

This study considers U as a class consisting of functions w which are analytic in an open unit disk in the complex plane. Let S be the subclass of U consisting of univalent functions and normalized. If a function $f \in S$, then f has a Maclaurin series expansion. By making use of the principle of subordination, the subclass of convex functions with respect to symmetric conjugate points is introduced. The coefficient estimates are determined by using the method of mathematical induction. The distortion theorem and integral operator are obtained for functions in the new class.

SUBKELAS BAGI FUNGSI CEMBUNG TERHADAP TITIK SIMETRI KONJUGAT

ABSTRAK

Kajian ini mempertimbangkan U sebagai kelas yang terdiri daripada fungsi w yang analisis di dalam cakera unit terbuka pada satah kompleks. Andaikan S merupakan subkelas bagi U yang terdiri daripada fungsi univalen dan ternormal. Jika suatu fungsi $f \in S$, maka f mempunyai suatu kembangan siri Maclaurin. Dengan mengaplikasikan prinsip subordinasi, subkelas bagi fungsi cembung terhadap titik simetri konjugat telah diperkenalkan. Anggaran pekali ditentukan dengan menggunakan kaedah induksi matematik. Teorem herotan dan operator pengkamir turut diperoleh bagi fungsi di dalam kelas yang diperkenalkan.

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LIST OF SYMBOLS

$\leq$	less than or equal to
$\geq$	greater than or equal to
$<$	less than
$>$	greater than
$\prec$	subordinate to
$\in$	an element of
$\subseteq$	subset of or equal to
$\rightarrow$	approach to
!	factorial
$\bullet$	multiplication
$\bar{z}$	conjugate z
$\sum$	summation
$\prod$	multiplication
*	Hadamard product

CHAPTER 1

INTRODUCTION

1.1 Introduction

This study considers the class of analytic functions w in the open unit disk $D = \{z: |z| < 1\}$ of the form

$$w(z) = \sum_{k=1}^{\infty} b_k z^k$$

and satisfying the condition $w(0) = 0$ and $|w(z)| < 1, z \in D$ which is denoted by U. Let S be the subclass of U consisting of univalent functions and satisfying the normalized condition $f(0) = f'(0) - 1 = 0$. In Goodman (1975), a function f is univalent in D if it gives one-to-one mapping onto its image, $f(D)$. In other words, $f(z_1) = f(z_2)$ implies that $z_1 = z_2$, for $z_1, z_2 \in D$.

Univalent functions are also known as '*shclicht*' in German word means simple and '*odnolistni*' in Russian word means single-sheeted. Many researchers have shown their interests in this topic. Spencer (1947) is one of those mathematicians involved in this field.

If $f \in S$, then f has a Maclaurin series expansion of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

where a_n is a complex number.

There are four main subclasses of S which are starlike, convex, close-to-convex and quasi-convex. However, only the starlike and convex functions will be focused in our study.

1.2 Starlike Function

Definition 1.2.1 (Goodman, 1975) A set E, $E \subseteq C$ in the complex plane is said to be starlike with respect to w_0 an interior point of E if each ray with initial point w_0 intersects the interior of E in a set that is either a line segment or a ray. If a function $f(z)$ maps D onto a domain which is starlike with respect to w_0, then $f(z)$ is said to be starlike with respect to w_0. In the special case that $w_0 = 0$, $f(z)$ is a starlike function.

An example of starlike domain is shown in Figure 1.1 by its geometrical representation.

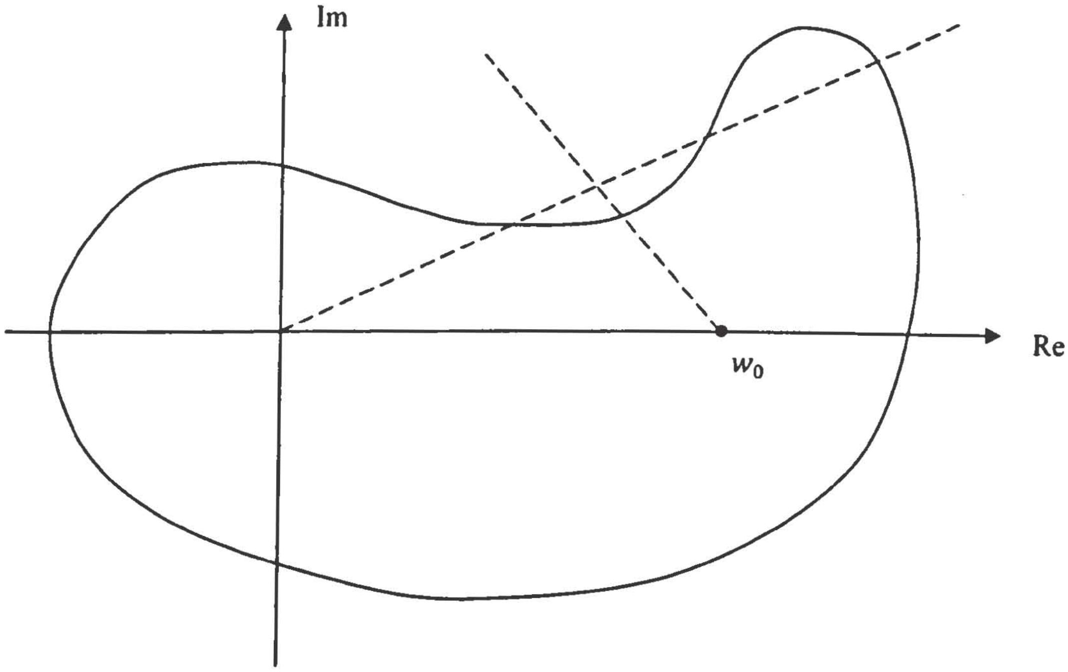

Figure 1.1 Starlike domain
(Source from Goodman, 1975)

The class of starlike functions is denoted by S^*. An analytic description of the functions $f \in S^*$ is then provided.

Definition 1.2.2 (Goodman, 1975) Let f be analytic in D with the condition $f(0) = f'(0) - 1 = 0$. Then, $f \in S^*$ if and only if

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0, \qquad z \in D. \tag{1.2}$$

1.3 Convex Function

Definition 1.3.1 (Goodman, 1975) A set E in the complex plane is called convex if for every pair of points w_1 and w_2 in the interior of E, the line segment joining w_1 and w_2 is also in the interior of E. If a function $f(z)$ maps D onto a convex domain, then $f(z)$ is called a convex function.

An example of convex domain is shown in Figure 1.2 by its geometrical representation.

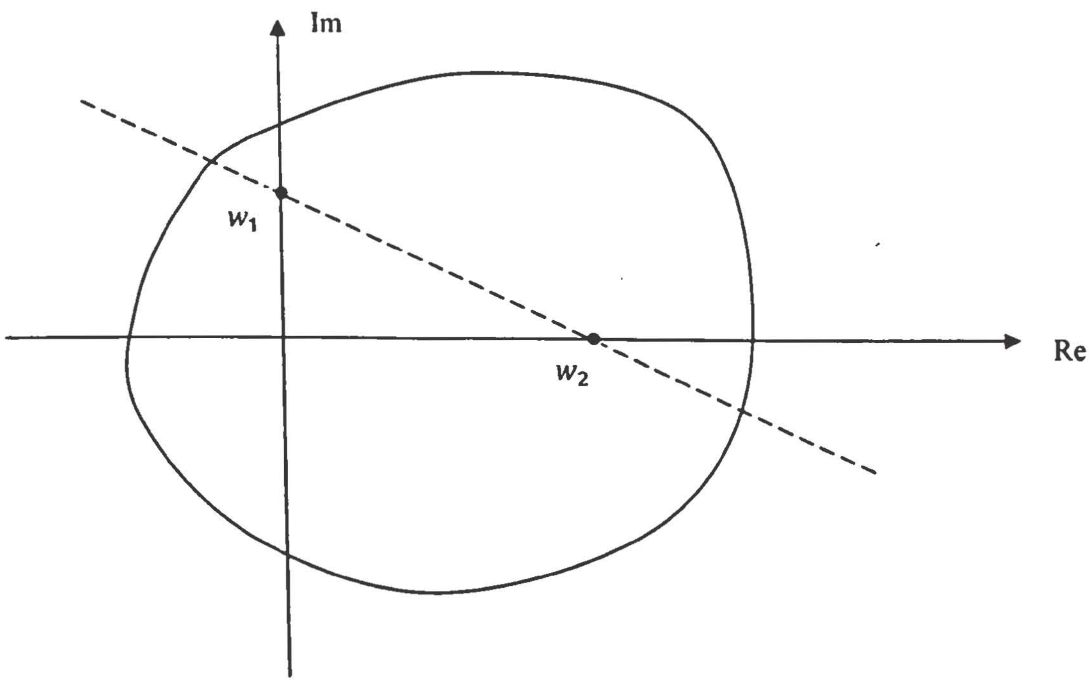

Figure 1.2 Convex domain

(Source from Goodman, 1975)

Let C denote the class of convex functions. An analytic description of the functions $f \in C$ is then provided.

Definition 1.3.2 (Goodman, 1975) Let f be analytic in D with the condition $f(0) = f'(0) - 1 = 0$. Then, $f \in C$ if and only if

$$Re\left\{\frac{(zf'(z))'}{f'(z)}\right\} > 0, \qquad z \in D. \tag{1.3}$$

1.4 Subordination

In this study, the concept of subordination is also considered. The definition of subordination is as follows:

Definition 1.4.1 (Miller & Mocanu, 2000) Assume that $f, g \in U$. The functions f is subordinate to g and written as $f \prec g$, if there exists a function $w \in U$ such that $f(z) = g(w(z))$ for all $z \in D$. If g is univalent in D, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(D) \subseteq g(D)$.

1.5 Objectives of Study

There are four objectives to be achieved in this study. The objectives are;

a. to introduce the subclass of convex functions with respect to symmetric conjugate points, which is denoted by $C_{sc}(A,B), -1 \leq B < A \leq 1$;

b. to determine the coefficient estimates for functions $f \in C_{sc}(A, B)$;

c. to determine the distortion theorem for functions $f \in C_{sc}(A, B)$; and

d. to determine the integral operator for functions $f \in C_{sc}(A, B)$.

1.6 Scope of Study

In this study, the subclass of convex functions with respect to symmetric conjugate points will be focused. By considering the class of convex functions with respect to symmetric conjugate points and using the ideas from Goel & Mehrok (1982), the new class of functions which is denoted by $C_{sc}(A, B), -1 \leq B < A \leq 1$ will be introduced. The coefficient estimates, distortion theorem and integral operator for the functions $f \in C_{sc}(A, B)$ will be determined.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The class of starlike and convex functions with respect to symmetric, conjugate and symmetric conjugate points will be discussed in this chapter. The class of starlike functions with respect to symmetric points was first introduced by Sakaguchi (1959). In the later years, the class has also been studied by Robertson (1961), Stankiewicz (1965), Owa (1984) and Wu (1987). In the year 1987, El-Ashwah & Thomas introduced two other classes namely the class of starlike functions with respect to conjugate and symmetric conjugate points.

2.2 Starlike With Respect To Symmetric, Conjugate and Symmetric Conjugate Points

The class of starlike functions with respect to symmetric points was initially introduced by Sakaguchi (1959). Let $f(z)$ be a function which is analytic in D, and suppose that for every $r \to 1 (r < 1)$ and every z_0 on $|z| = r$, the angular velocity of the function $f(z)$ about the point $f(-z_0)$ is positive at $z = z_0$ as z traverses the circle $|z| = r$ in the positive direction. According to Sakaguchi (1959), the function f is said to be starlike with respect to symmetric points.

Figure 2.1 shows the geometrical representation of starlike with respect to symmetric points domains.

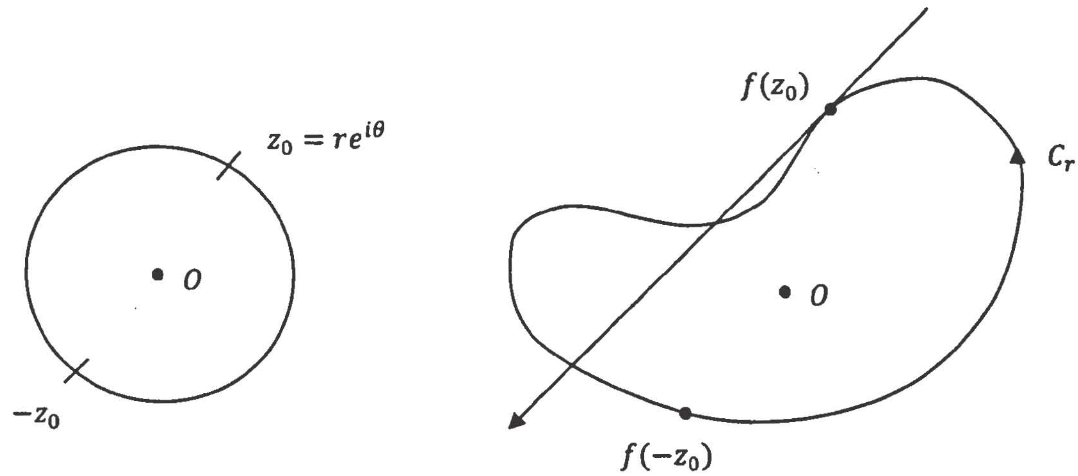

Figure 2.1 Starlike with respect to symmetric points
(Source from Sakaguchi, 1959)

Sakaguchi (1959) also proved that the condition is equivalent to the following definition.

Definition 2.2.1 (Sakaguchi, 1959) Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is starlike with respect to symmetric points if and only if

$$Re\left\{\frac{zf'(z)}{f(z) - f(-z)}\right\} > 0, \quad z \in D. \tag{2.1}$$

This class is denoted by S_s^*.

El-Ashwah & Thomas (1987) introduced two other classes namely starlike functions with respect to conjugate and symmetric conjugate points. The definitions for the two other classes are given as follows:

Defnition 2.2.2 (El-Ashwah & Thomas, 1987) Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is starlike with respect to conjugate points if and only if

$$Re\left\{\frac{zf'(z)}{f(z) + \overline{f(\bar{z})}}\right\} > 0, \qquad z \in D. \tag{2.2}$$

This class is denoted by S_c^*.

Definition 2.2.3 (El-Ashwah & Thomas, 1987) Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is starlike with respect to symmetric conjugate points if and only if

$$Re\left\{\frac{zf'(z)}{f(z) - \overline{f(-\bar{z})}}\right\} > 0, \qquad z \in D. \tag{2.3}$$

This class is denoted by S_{sc}^*.

2.3 Convex With Respect To Symmetric, Conjugate and Symmetric Conjugate Points

Das & Singh (1977) introduced the class of convex functions with respect to symmetric points. The analytic representation of the class is given in the following definition.

Definition 2.3.1 (Das & Singh, 1977) Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is convex with respect to symmetric points if and only if

$$Re\left\{\frac{(zf'(z))'}{(f(z) - f(-z))'}\right\} > 0, \qquad z \in D. \tag{2.4}$$

This class is denoted by C_s.

The classes C_c and C_{sc} can be further extended from the class C_s.

Definition 2.3.2 Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is convex with respect to conjugate points if and only if

$$Re\left\{\frac{(zf'(z))'}{(f(z) + \overline{f(\bar{z})})'}\right\} > 0, \qquad z \in D. \tag{2.5}$$

This class is denoted by C_c.

Definition 2.3.3 Let f be analytic in D with $f(0) = f'(0) - 1 = 0$. Then, f is convex with respect to symmetric conjugate points if and only if

$$Re\left\{\frac{(zf'(z))'}{(f(z) - \overline{f(-\bar{z})})'}\right\} > 0, \qquad z \in D. \tag{2.6}$$

This class is denoted by C_{sc}.

2.4 Class $S_s^*(A, B)$ and $S_c^*(A, B)$

Goel & Mehrok (1982) introduced a subclass of S_s^* denoted by $S_s^*(A, B)$. Let $S_s^*(A, B)$ be the class of functions of the form (1.1) and satisfying the condition

$$\frac{2zf'(z)}{f(z) - f(-z)} \prec \frac{1 + Az}{1 + Bz}, \qquad -1 \leq B < A \leq 1, \qquad z \in D. \tag{2.7}$$

By definition of subordination, it follows that $f(z) \in S_s^*(A, B)$ if and only if

$$\frac{2zf'(z)}{f(z) - f(-z)} = \frac{1 + Aw(z)}{1 + Bw(z)} = P(z), \qquad w(z) \in U \tag{2.8}$$

where

$$P(z) = 1 + \sum_{n=1}^{\infty} p_n z^n. \tag{2.9}$$

Mad Dahhar & Janteng (2009) introduced a subclass of S_c^* denoted by $S_c^*(A,B)$. Let $S_c^*(A,B)$ be the class of functions of the form (1.1) and satisfying the condition

$$\frac{2zf'(z)}{f(z)+\overline{f(\bar{z})}} \prec \frac{1+Az}{1+Bz}, \qquad -1 \le B < A \le 1, \qquad z \in D. \tag{2.10}$$

By definition of subordination, it follows that $f(z) \in S_c^*(A,B)$ if and only if

$$\frac{2zf'(z)}{f(z)+\overline{f(\bar{z})}} = \frac{1+Aw(z)}{1+Bw(z)} = P(z), \qquad w(z) \in U \tag{2.11}$$

where $P(z)$ is given by (2.9).

2.5 Class $C_s(A,B)$

Janteng & Abdul Halim (2008) introduced a subclass of C_s denoted by $C_s(A,B)$. Let $C_s(A,B)$ be the class of functions of the form (1.1) and satisfying the condition

$$\frac{2(zf'(z))'}{(f(z)-f(-z))'} \prec \frac{1+Az}{1+Bz}, \qquad -1 \le B < A \le 1, \qquad z \in D. \tag{2.12}$$

By definition of subordination, it follows that $f(z) \in C_s(A,B)$ if and only if

$$\frac{2(zf'(z))'}{(f(z)-f(-z))'} = \frac{1+Aw(z)}{1+Bw(z)} = P(z), \qquad w(z) \in U \tag{2.13}$$

where $P(z)$ is given by (2.9).

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