

EXPLOSION SIMULATION USING SPHERICAL COORDINATES SYSTEM

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DECLARATION

I declare that this thesis is the result of my own work, except the quotations, equations and summaries, each of which the sources has been mentioned.

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Abstract

Explosion simulation had been a common scene whether it has been used for scientific research or entertainment industry. Various approaches had been taken in order to bring out the most ideal result. The primary concerned of this project is on the calculation of displacement by using the spherical coordinates system. Thorough explanation was given along with the appropriate illustration such as the system architecture and UML diagram. Pseudo code and the end results were provided together to emphasis on the feasibility of the simulation. Furthermore, experiments were conducted on the system; discussion and detail analysis were shown based on the result obtained from the experiment. From the experiment, the number of debris model used has direct impact on the system performance. The increment in the debris model is directly proportional to the system run time and frames per second. The default number of debris model used in the system is 14600; in the experiment 75020 debris model was used. The increase of almost 5 times in debris model, increase the system run time by 5 times and the drop of frames per second by 5 times as well. A conclusive summary was given to highlight the contribution being done and the possible for future works.

SIMULASI LETUPAN MENGGUNAKAN SISTEM KOORDINAT SFERA

Abstrak

Simulasi letupan telah banyak digunakan dalam penyelidikan saintifik atau industri hiburan. Objektif utama projek ini adalah mengenai pengiraan sesaran dengan menggunakan sistem koordinat sfera. Penjelasan yang mendalam telah diberikan bersama dengan ilustrasi yang sesuai seperti seni bina sistem dan gambar rajah UML. Selain itu, eksperimen telah dijalankan pada system serta perbincangan dan analisis terperinci akan ditunjukkan berdasarkan keputusan yang diperolehi daripada eksperimen tersebut. Berdasarkan hasil eksperimen, bilangan model yang digunakan mempunyai kesan langsung pada prestasi sistem. Peningkatan dalam model serpihan adalah berkadar secara langsung dengan jangka masa sistem dan "frame per second". Pada sistem tersebut ia mempunyai 14600 model; dalam eksperimen tersebut sebanyak 75020 model telah digunakan. Penambahan 5 kali pada bilangan model, telah memanjangkan jangka masa system masa 5 kali ganda dan penurunan "frame per second" sebanyak 5 kali ganda. Kesimpulan terhadap sistem tersebut akan dibincangkan bersama dengan peluasan kajian.

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CHAPTER 1

INTRODUCTION

1.1 Preamble

Explosion, like many other natural phenomena it is awe-inspiring yet destructive. Human had learned to respect and fear nature along the course of history, from 1556, Shaanxi earthquake in China to 2011 Tsunami that strike Japan. The most common occurrence of explosion that can be seen in the nature is the eruption of volcanoes, some explosion can even happen far in the universe such as the explosion of a star known as supernova. There are human-made explosion like the explosion of gunpowder and combustion of the engine. Gunpowder or any explosive material can be used in the mining process; the most notable event would be the industrial revolution in United Kingdom. Gradually, explosion is being introduced in the usage of warfare; one of the most shocking incidents is the bombing of Hiroshima and Nagasaki in Japan during World War II. It is clearly that the role of explosion had been significant throughout the human civilization, from the ancient Greece toward the modern society.

In (Ligomenides, 2009), it introduced the working mechanism of human mind can be similar to the working of mathematical theory like quantum mechanics. It further press on that human mind has the ability to reason and justify a situation. However there is a contradicting issues which stated in the study, for example the force of gravity, is this particular force already been there at the first place waiting for human to discover or it is just human mere perception to justify what seem to be reasonable. This study suggested that in the physical world where human live in; mathematics is capable of justify and explain all kinds of event happened whether is

act or science. For instance the study by (Marchi, 2007) stated the texture and taste of wine can be model using mathematical formulae, the result obtained from the study had been compared by using the scale of wine tasting by the sommelier to support the accuracy of the mathematical model. From these previous studies, the ability of human mind using mathematical approach in understand thus justify an event had help to figure out the process of explosion.

As the learning process on the working of explosion unfold, in order to learn more on it, the approach on simulating the event of explosion begun. With the aid of technology, the simulation of explosion can be done ranged from static image to sophisticated graphics rendering. During the simulation process of explosion, it is important to understand the characteristic or working behavior to better present a accurate and realistic result especially in the field of scientific research. Nowadays, the simulation of explosion is no longer the sole purpose of science and research. In the entertainment industry such as movie, advertisement and computer games need it to create a better viewing experience. In (Kruger, 2005) stated the rapid growth in the virtual environments and computer games, required a much more realistic simulation and real time rendering process that enhance the feeling of immersion and vividness.

In the event of explosion simulation, few common techniques had been applied such as the meteorite simulation using particle system in (Wang *et al.*, 2010), computational fluid dynamics in simulation of dust explosion (Collecutt *et al.*, 2009), using connected-voxel in exploding object (Mazarak *et al.*, 1999) and vortex particle for explosion (Selle *et al.*, 2005). Each new technique was intended to create a better approach in simulation and to cover up the limitation of other techniques. However each of this technique was not perfect and having its own potential disadvantages and advantages. The implementation of technique would rely on the objective or purpose of the study to obtain a desirable result.

A balance trade-off between the simulation process and the technique implemented has to take into the consideration of the scope and limitation of the study. For example, while conducting a control experiment, only the concern

variables are taken into account, other negligible factors need to be trimmed out and focus on the objective.

1.2 Problem Background

In the field of computer graphics especially in the event of scientific study, it required careful evaluation of the data to make sure the rendering process would not be distorted. Unlike in the entertainment industry that emphasizes on the viewing experience; a genuine and realistic rendering result needs to be based on mathematical formulae and physics theory. Adapting the scientific approach would lower the risk of data redundancy and systematic error.

Many previous studies had been done on the simulation of explosion; however most of them focus on the after effect of the explosion such as the fire and smoke instead of the debris or after blast particles. This argument was introduced in (Mazarak *et al.*, 1999), it argued that the market itself tends to focus more on the simulation of fire and smoke. Furthermore it stresses on that the study in the “debris modeling” seems to be abandoned by researchers. The “debris modeling” emphasizes on the displacement of the exploding object, for example when an object is being exploded, where the object would be after the blast. Each study has its own limitation and restriction, it is impossible to simulate the event of explosion that accounts for every detail. The perfect explosion simulation system is yet to be created; in (Mazarak *et al.*, 1999) the purpose is to fill the gap in the field of explosion simulation.

The different focus in each study leads to different techniques being applied, there are studies that deal heavily on the calculation of data such as the range of blast wave in (Takahashi *et al.*, 2010). Usually this kind of study would adapt the computational fluid approach, by doing so ensure the accuracy and precision of the data. However this technique required expensive computational time, a similar issue was stated in (Jiang *et al.*, 2001).

There are studies that did not cover every aspect on the simulation process which make the whole process much easier to be done, however the degree of realism would be lower. The common approach is using the particle system, where the visual perception is much more focus. For example in (Feldman *et al.*, 2003), it did not drill on the blast wave calculation, instead it focuses on the effect of after blast in term of smoke and fire.

There is no absolutely right or wrong in both cases, as each have different target to achieve. In the course of this project, the simulation process will be focus on the displacement of the blasted particles.

1.3 Problem Statement

Explosion simulation that focuses on the "debris modeling" is strongly affected by two main factors which included the characteristic of the blast waves and any physics theory that may deem relevant.

1.4 Aim

To create an explosion simulation that able to bring forth the "debris modeling".

1.5 Objectives

For the aim to be achieved, several studies have to be done beforehand.

- I. To generate the "debris model" using primitives shape.
- II. To develop a prototype simulation system by using the spherical coordinates system.

1.6 Scope of Study

- I. The explosion simulation will only focus on the blast wave effect against the blasted object.
- II. The explosion simulation does not involved the generation of any after blast effect such as smoke and gas, or any other related events.
- III. The explosion simulation does not take into consideration of external factor such as wind speed and environment.

1.7 Justification

The approach of implementing physics theory and mathematics formulae in the simulation process of explosion would broaden the scope of study. As most of the works created an illusion effect of replacing part of the explosion process by fire or smoke, without precisely present the displacement of the exploding object.

The approach of this project was not to introduce new technique; rather it is a refresh on the previous work done by other researchers and being able to bring forth a new working ideas. The work of a man alone can never out run the massive growing civilization; new thing have been add in even in the library of programming language. Improvement and modification on the previous work will definitely be a stepping stone for the next researcher to come.

1.8 Thesis Organization

The organization of this project has six distinct chapters and an overview will be given on each chapter.

In chapter 1, a simple go through on the computer graphics working trends and the expansion of such industry. The objectives and aim will be highlighted; the working scope will clarify according to the problem background. The problem

statement serves as a working guideline in this project and justification will be given to support it.

In chapter 2, an overview on the history of explosion will be explained. Previous works and researches will be highlighted as it narrow down the possible improvement needed in each approach.

In chapter 3, a thorough working method will be explained along with suitable graphical illustration. It explained on the theoretical approach of the spherical coordinates system. The modeling representation will be discussed as well.

In chapter 4, the system architecture and working structure will be explained on the designing level. An in-depth working mechanism on the program provided along with UML diagram to support the feasibility of the system.

In chapter 5, the implementation of the spherical coordinates system in the system will be discussed. Pseudo code will be given along with the implementation of the formulae. Experiment will be conducted on several aspects to test its strength and weakness, thus open the possibility of optimization.

In chapter 6, conclusion will be stated on the system and personal contribution will be highlighted. Possible future works and enhancement of the system will be discussed.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The explosion process itself involved several stages and each kind of explosion varied among each other. Simulation of smoke, fire and dust are the most common way being implemented to represent the aftermath of explosion. By simulating these immediate effect of explosion, had greatly enhance the viewing experience of the user. However by doing so the mechanism behind of the process of explosion such as the acceleration and velocity might be neglected. The whole simulation process itself might just look like a fancy trick of illusion, in the strictest term it is more like animating rather than simulating. The reason of saying so is that most research works did not focus on the “invisible string” that happened during the explosion which is the blast wave. The main contributing factor that lead to the fracture or even the demolish of the object is caused by the blast wave; the sudden rapid changes of pressure by the blast wave upon the object would eventually breakdown the whole structure of the object.

In this study, it is essential to understand the displacement and structure of the object after getting the full impact by the blast wave. Finally, the discussions of few common rendering techniques that have to be evaluated.

2.2 History

From the prehistoric to the modern society, explosion never stops from occurring regardless of place and time, it may carry out in different forms but they were all “shining spackle” that cannot be easily ignored. Behind the many craftworks by nature such as thunder, explosive volcanic eruptions even meteorite impact is all root from the family of explosion.

Before civilization was formed, God or any divine being was the common explanation on these nature phenomena. Human were being superstitious and ignorant toward the logical thinking of deduction and justification. However the mind of human being know no bound, in time they were able to seek pattern in nature thus suggesting that behind the many natural phenomena, some can be explained without the involvement of any divine being. For example, people learn that the happening of eclipse is actually a govern pattern of the orbit. Even without the ritual of worshipping some pagan God, the sun and moon will soon emerge from the shadow (Hawking, 2010).

After fast forward a few hundred years, at the 17th century, a revolutionary breakthrough emerges in the field of explosive material. The discovered and usage of gunpowder or the famous “black powder”; one of the most notable scientists Alfred Nobel has given a quote on the gunpowder. “Gunpowder, that old mixture, possesses a truly admirable elasticity which permits its adaptation to purposes of the most varied nature. Thus, in a mine it is wanted to blast without propelling; in a gun to propel without blasting; in a shell it serves both purposes combined; in a fuse, as in fireworks, it burns quite slowly without exploding. But like a servant for all work, it lacks perfection in each department, and modern science armed with better tools, is gradually encroaching on its old domain.” (Krehl, 2009)

From the course of history itself, it can be seen that explosion come in various form. By far, there are no absolute or the “whole package” form of simulation technique, each technique was created to handle different scope, limitation and objective. Given a glimpse through the long road of history was just to broaden the

idea of explosion, where this field vast in nature and each single branch would lead to a new approach of study.

2.3 Mathematics Formulae and Physics Theory

2.3.1 Projectile Motion

Physics is the study of the physical world, we all live in. The purpose of physics is to understand the rules or laws that govern everything. Physics used to give the impression of being complicated as one of the most notable scientist such as Albert Einstein and Stephen Hawking discussed on the complex issues like atomic particle and theory of relativity. Actually physics is something close to us, and most of the time, physics theory is being applied without knowing it. For example the faster a boxer gives out a punch, the harder it hit on the opponent; this is because generation of force rely on mass and acceleration. Most people may not ask the question behind every single action they did in their daily life, but one who study would try to answer these questions such as why a ship as large as "Titanic" can float on the water.

During the simulation process of explosion, the properties of displacement of the exploding object need to specify. Since the physics is all about the laws of physical world, it would deem appropriate to apply the certain physics theory to better represent the simulation process. The projectile motion discussed on the displacement of object when it was being launch or projected. This is similar to the event of explosion, as the explosion occurs, the rapid change of pressure would create immense force on the object, thus propelling the object or fracture its structure.

Projectile motion only focus on the movement of the projected object, it involved the consideration of gravity, velocity, and time, other external factor such air resistance and friction will not be discussed. The reason is almost every physics theory work like a controlled experiment, where the experiment involving the manipulation of variable or factor. Therefore only the related or main factor will take

into consideration, other negligible factor will be avoided. Hence in the explanation of projectile motion will only drill on the ideal cases. In ideal cases, the time taken for a projected object to launch to its peak and time taken for it to fall down is the same. In Figure 2.1, the shape of the projectile motion in ideal cases would be a perfect and symmetrical parabolic whereas in reality the shape would be alter because of the air resistant.

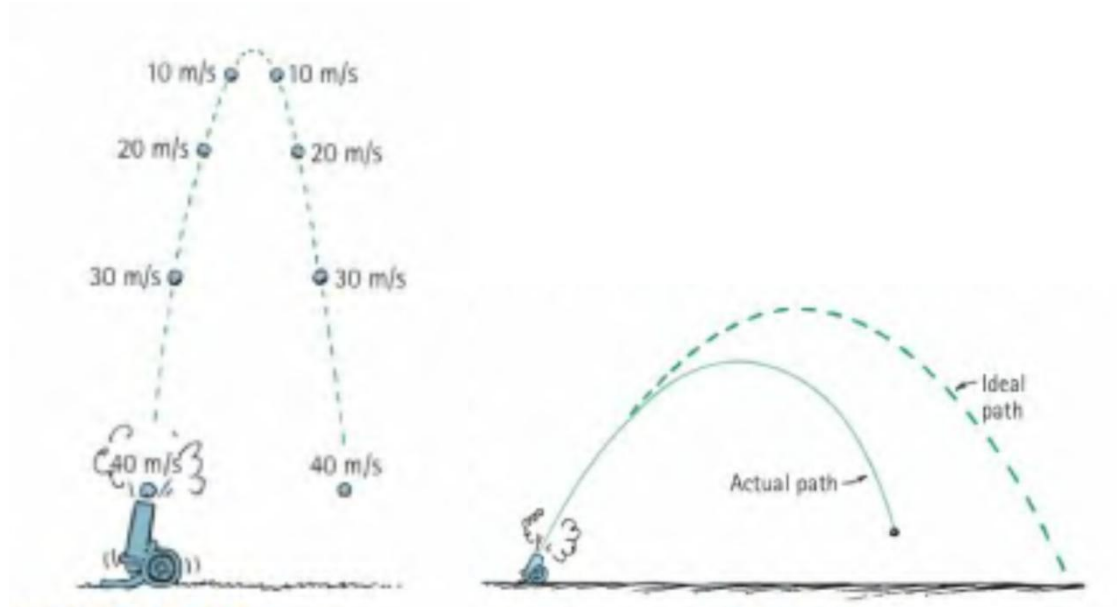


Figure 2.1 Projectile (Hewitt, 2010)

In Figure 2.2, the calculation of the displacement can be divided into two components, the vertical and the horizontal. The actual path for the projectile is the combination of both forces, which is the vector summation of both horizontal and vertical. The vertical force that pull the object upward and the horizontal force that pull the object across the horizontal level, would lead to the formation of a parabolic shape pulling force.

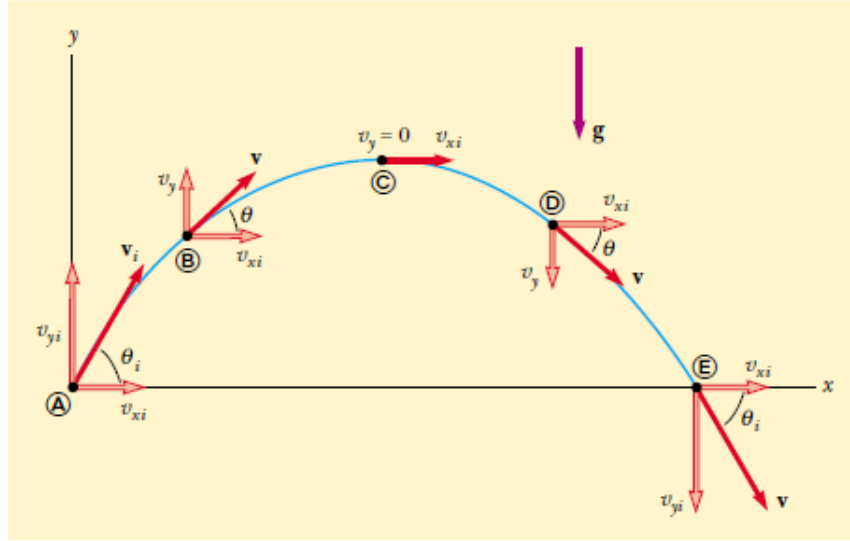


Figure 2.2 Projectile (Serway, 2004)

The general formulae for projectile motion:

$$v_x = v_i(\cos\theta), v_y = v_i(\sin\theta) \quad (2.1)$$

$$\Delta x = v_i(\cos\theta)\Delta t \quad (2.2)$$

$$v_y = v_i(\sin\theta) - g(\Delta t) \quad (2.3)$$

$$(v_y)^2 = v_i^2(\sin\theta)^2 - 2g(\Delta y) \quad (2.4)$$

$$\Delta y = v_i(\sin\theta)\Delta t - \left(\frac{1}{2}\right)g(\Delta t^2) \quad (2.5)$$

2.3.2 Blast Wave Theory

The approach of this project highlighted on the blast wave created by the explosion. The fundamental understanding of blast wave theory is crucial while modelling the object in the simulation. However in most of the research works, the blast wave theory was not implemented in it. In (Mazarak *et al*, 1999), it uses voxel to model the debris and fragments in the explosion. Each voxel had predefined characteristic based on the blast wave theory.

In (Yngve *et al*, 2000), a much better improvement in the blast wave theory model was introduced by the merging with the theory fluid dynamics. In (Yngve *et al*, 2000), it adds in more visual effect such as the fire and smoke yet without

neglecting the blast wave theory. In (Feldman *et al.*, 2003), it did a similar as (Yngve *et al.*, 2000), the difference between them is that (Feldman *et al.*, 2003) did not make much focus and improvement on the blast wave. The model it presented focusing on modelling the flame after the explosion, to make the flame itself much more realistic and vivid. It seems that the research of blast wave theory on explosion simulation came to a halt at the early of 21st century.

Blast wave, a shock wave in air that emitted from an atmospheric explosion, it is accompanied by a strong wind as felt by the observer when the wave passes by. (Krehz, 2009). This is the general idea of blast wave, when the blast wave happened; one could feel the changes of pressure around it given accompanied by a gust of wind. In a more scientific sense, blast wave is a shock wave which decays immediately after the peak is reached. This decay occurs in all variables including: pressure, density and material velocity. The rate of decay is, in general, different for each of the parameters.(Needham, 2010). Therefore the formation of blast wave is not ever lasting; the increment to the peak of blast wave is almost instantaneous. But in order for the surround the pressure return to its ambient or normal level, the decay of blast wave occurs right after it reaches the peak.

The Friedlander equation was well-known in the modelling of blast wave because of its simplicity and accurately predicts the shape of the blast wave. The Friedlander equation only involved two variable, time (t) and pressure (p). (Dewey, 2010)

$$P = P_s e^{-\frac{t}{t^+}} \left(1 - \frac{t}{t^+} \right) \quad (2.6)$$

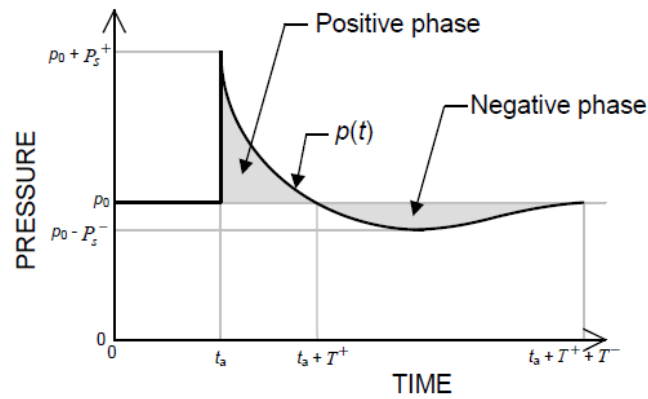


Figure 2.3 Friedlander Blast Wave Equation Graph (Martins *et al.*, 2001)

In Figure 2.3, P_0 is the ambient pressure, which means the surrounding or normal pressure; in general it is 1 atmospheric pressure (atm). t_a is the time where the blast wave reaches its peak, where the highest pressure caused by the blast wave. The positive phase is where the pressure caused by the blast wave is higher than the ambient pressure. $t_a + T^+$ is the time where the decay of blast wave reaches the ambient pressure. However the decay of blast wave will still go on where it is called the negative phase, in negative phase the pressure is lower than the ambient pressure. $t_a + T^+ + T^-$ is the total time needed for the pressure to return to its ambient pressure.

However the Friedlander equation only applicable under the limitation of atmospheric pressure equal to one. There a Modified Friedlander equation was introduced, with slight modification of the original Friedlander equation by adding a new coefficient to counter the situation of different atmospheric pressure. (Dewey, 2010)

$$P = P_s e^{-\alpha t} \left(1 - \frac{t}{t^+}\right) \quad (2.7)$$

2.4 Techniques

2.4.1 Particle System

The most notable work done on explosion simulation was (Reeves, 1983), in his work of particle system. There are three characteristic in particle system.

- I. The object itself is not represented by primitives such as triangle or line, it had no specific type or form, is a cloud of primitive which can be defined later on.
- II. The particle system is not a static entity, the form and movement changes as time goes. Some particle can be created or destroyed along the process to represent fuzzy nature in explosion.
- III. To handle the variation and randomness in the event of explosion, the particle system is not predefined; stochastic process was used to change form, appearance or location of the particle.

The algorithm of particle generation per fame, f is

$$NParts_f = MeanParts_f + Rand() * VarParts_f \quad (2.8)$$

where $Rand()$ is the generation of random number between -1.0 and +1.0, $MeanParts_f$ is the mean number of particles, and $VarParts_f$ is the variance of the particles

The algorithm of particle gereneration per frame, f based on the screen size is,

$$NParts_f = (MeanParts_{saf} + Rand() * VarParts_{saf}) * ScreenArea \quad (2.9)$$

where Mean is the mean number of particles generated, $MeanParts_{saf}$ is the mean per screen area, $VarParts_{saf}$ is the variance per screen area and $ScreenArea$ is the particle system's screen area.

The algorithm for the particle system to modify in intensity,

$$MeanParts_f = InitialMeanParts + DeltaMeanParts * (f - f_0) \quad (2.10)$$

or

$$MeanParts_{saf} = InitialMeanParts_{sa} + DeltaMeanParts_{sa} * (f - f_0) \quad (2.11)$$

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