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A simple approach to solving the kinematics of the 4-UPS/PS *(3R1T)* parallel manipulator†

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Abstract

This work reports on the position, velocity and acceleration analyses of a four-degrees-of-freedom parallel manipulator, 4-DoF-PM for brevity, which generates Three-rotation-one-translation *(3R1T)* motion. Nearly closed-form solutions to solve the forward displacement analysis are easily obtained based on closure equations formulated upon linear combinations of the coordinates of three non-collinear points embedded in the moving platform. Then, the input-output equations of velocity and acceleration of the robot manipulator are systematically established by resorting to the theory of screws. To this end, the Klein form of the Lie algebra *se(3)* of the Euclidean group *SE(3)* is systematically applied to the velocity and reduced acceleration state in screw form of the moving platform cancelling the passive joint rates of the parallel manipulator. Numerical examples, which are confirmed by means of commercially available software, are provided to show the application of the method.

Keywords: Parallel manipulator; Semi-closed form solution; Klein form; Lie product; Acceleration analysis; Screw theory

1. Introduction

Simpler kinematics and control are the main attributes of parallel manipulators with fewer than six degrees of freedom, commonly known as defective parallel manipulators, when these are compared with the classical hexapod introduced by Gough and Whitehall [1, 2]; e.g., while closed and nearly closed-form solutions to solve the forward displacement analysis are available for most defective parallel manipulators, the derivation of a closed-form solution concerned with the forward displacement analysis of the hexapod is perhaps an unrealistic task. Furthermore, it is noticeable the growing acceptance of defective parallel manipulators in both academia and industry due to their simplified designs demanding fewer links and actuators. In that trend, four-degrees-of-freedom parallel manipulators have found interesting applications such as laparoscopic surgery, ship's heave and swing motion simulation, robots for generating Schöenflies motion, hip joint simulator, pick-and-place robot, aeronautical devices and so forth. Several approaches have been recursively employed to elucidate the kinematics and dynamics of four-degrees-of-freedom parallel manipulators. In that way Altuzarra et al. [3] considered the dynamics of a parallel manipulator generator of Schöenflies motion based on Lagrangian formulation. Choi and Ryu [4] applied reciprocal-screw theory to perform the singularity analysis of the H4 robot [5]. Sheng et al. [6] applied the theory of screws to develop a class of four-degrees-of-freedom parallel manipulators with large rotational workspace. Dong et al. [7] introduced a docking equipment device for the aircraft industry in which the theory of screws is employed in order to approach the mobility and kinematic analyses of the parallel manipulator. Song et al. [8] reported the optimum performance of a two-legged four-degrees-of-freedom parallel manipulator by handling the reciprocal screw of a wrench on a screw, named the virtual power.

The four-degrees-of-freedom parallel manipulator generator of the *3R1T* motion is the motive of the contribution. A pioneering version of that robot is credited to Zlatanov and Gosselin [9] who in 2001 introduced a 3-RRRRR/RRC parallel manipulator following the trend of the Agile eye [10]: The inclusion of revolute joints with concurrent axes demanding precise conditions of assembly and manufacture. Li and Huang [11] employed the theory of screws in the topology synthesis of the 4-DoF-PM equipped with revolute joints, also demanding the strict condition of concurrent axes. Lu et al.

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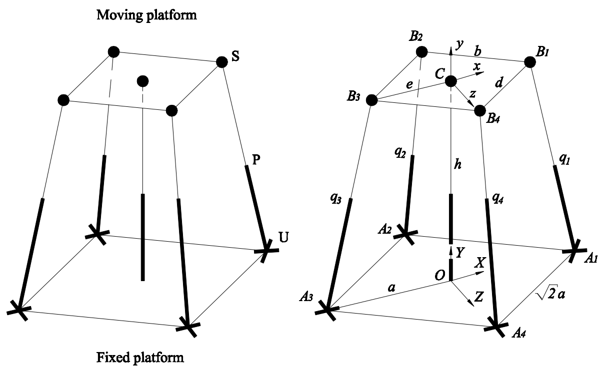

Fig. 1. 4-UPS/PS parallel manipulator.

[12] investigated the singularities of the 4-SPS/PS parallel manipulator by analyzing the Jacobians associated with decoupled motions. Meanwhile, Zhou and Peng [13] reported the singularity analysis of the same manipulator by resorting to the theory of screws.

In this work, the kinematics of the 4-UPS/PS parallel manipulator generator of the *3R1T* motion is approached by means of the theory of screws. As an intermediate step, the displacement analysis is achieved by applying a novel strategy which consists of formulating the closure equations of the robot upon simple linear combinations of the coordinates of three points embedded in the moving platform. Then, the input-output equations of velocity and acceleration of the parallel manipulator are established by resorting to reciprocal-screw theory. Note that the acceleration analysis does not require the computation of the passive joint acceleration rates of the robot under study. Numerical examples are provided to show the application of the method.

2. Topology of the robot manipulator

In this section the architecture of the parallel manipulator at hand is outlined. With reference to Fig. 1, the parallel manipulator under study comprises a rectangular moving platform p of sides b and d and a square fixed platform 0 of side $\sqrt{2}a$ connected to each other by means of four active identical UPS-type limbs and one passive PS-type central kinematic chain labeled as the fifth limb of the manipulator. Meanwhile, the circumferential prismatic joints are assumed to be the inputs, i.e., the linear actuators or generalized coordinates $q_i\left(i=1,2,3,4\right)$ of the parallel manipulator.

Let O-XYZ be a reference frame attached to the fixed platform whose origin O is located at the center of the fixed platform with the X-axis directed from O to point A_1 and the Y-axis normal to the plane of the fixed platform. Similarly, let C-xyz be a reference frame attached to the moving platform whose origin C, located by vector $\boldsymbol{C}$, is precisely the center of the moving platform while the x-axis points from C to point B_1 and the y-axis is normal to the plane of the moving platform. Moreover, point A_i, located by vector $\boldsymbol{A_i}$, is defined as the intersection of the perpendicular axes of two revolute joints simulating the universal joint of the ith surrounding limb. Meanwhile, the center of the spherical joint in the same limb is notated as point B_i located by vector $\boldsymbol{B_i}$. Note that point $C = (0,h,0)$ is the center of the spherical joint connecting the passive leg to the moving platform, where h stands for the extension of the passive prismatic joint, i.e., h is the signed distance between the origins of the two reference frames. The architecture of the robot is such that three rotational plus one translational freedoms are available for its moving platform. Evidently, the loss of mobility of the robot manipulator is due to the passive central kinematic chain. Although this action diminishes in general the performance of the robot, simpler kinematics and control are the main assets of the robot under study when compared with the general six-degrees-of-freedom parallel manipulator.

3. Displacement analysis

In this section the finite kinematics of the robot manipulator is presented. The forward displacement analysis comprises the computation of the pose of the moving platform as measured from the fixed platform given the generalized coordinates q_i. The strategy to address this problem is based on the generation of a 40th-order polynomial equation in the unknown h. To this end, let us consider that the fact that the pose of rigid body may be fully determined upon the coordinates of three points embedded to it has been well explored in order to solve the forward displacement analysis of parallel manipulators [14, 15]. Following that trend, let $B_1 = (w_1,w_2,w_3)$, $B_2 = (w_4,w_5,w_6)$ and $B_3 = (w_7,w_8,w_9)$ be *the control points of the robot.* Then, the coordinates of point B_4 may be formulated as a linear combination of the unknown coordinates of the control points accordingly the geometry of the moving platform as $B_4 = B_1 - B_2 + B_3$. Furthermore, taking into account that $C = (B_1+B_3)/2 = (0,h,0)$ then it follows that $w_7 = -w_1$ whereas $w_9 = -w_3$ and $w_8 = 2h - w_2$. With the purpose to express w_1 and w_2 in terms of the unknown h let us consider that two closure equations may be written upon the first and third legs as follows

$$\left(\boldsymbol{B}_i-\boldsymbol{A}_i\right)\cdot\left(\boldsymbol{B}_i-\boldsymbol{A}_i\right)-\left(\boldsymbol{B}_i-\boldsymbol{C}\right)\cdot\left(\boldsymbol{B}_i-\boldsymbol{C}\right)=q_i^2-e^2, \tag{1}$$

where the dot (.) denotes the inner product of usual three-dimensional vectorial algebra. Meanwhile, evidently $e^2 = \left(b^2+d^2\right)/4$. Solving Eq. (1) one obtains

$$w_1=\frac{4a^2+b^2+d^2-2q_1^2-2q_3^2+4h^2}{8a}, \tag{2}$$

and

$$w_2=\frac{q_1^2-q_3^2+4h^2}{4h}. \tag{3}$$

It is straightforward to show that the unknowns w_5 and w_6 are obtained similarly upon the second and fourth limbs. Once the coordinates w_1, w_2, w_5, w_6, w_7, w_8 and w_9 are expressed in terms of the unknown *h,* three compatibility kinematic constraint equations may be written as

$$(\boldsymbol{B}_1 - \boldsymbol{B}_2)\cdot(\boldsymbol{B}_1 - \boldsymbol{B}_2) = b^2, \tag{4}$$

$$(\boldsymbol{B}_3 - \boldsymbol{B}_2)\cdot(\boldsymbol{B}_3 - \boldsymbol{B}_2) = d^2, \tag{5}$$

$$(\boldsymbol{B}_1 - \boldsymbol{B}_3)\cdot(\boldsymbol{B}_1 - \boldsymbol{B}_3) = b^2 + d^2. \tag{6}$$

Expressions Eqs. (4)-(6) yield a non-linear system of three equations in the unknowns w_3, w_4 and h named *the characteristic equations of the parallel manipulator* given by

$$\begin{aligned} &a_1h^6 + (b_1 + c_1w_3 + d_1w_4)h^4 + \\ &(e_1 + f_1w_3 + g_1w_4 + i_1w_3^2 + j_1w_4^2)h^2 + k_1 = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} &a_2h^6 + (b_2 + c_2w_3 + d_2w_4)h^4 + \\ &(e_2 + f_2w_3 + g_2w_4 + i_2w_3^2 + j_2w_4^2)h^2 + k_2 = 0, \end{aligned} \tag{8}$$

$$a_3h^6 + b_3h^4 + (c_3 + d_3w_3^2)h^2 + e_3 = 0. \tag{9}$$

Therein the coefficients affecting the unknown h are computed upon the parameters and generalized coordinates of the parallel manipulator. To solve Eqs. (7)-(9), first w_3 is directly computed upon Eq. (9). After squaring selectively the remaining two equations and suppressing the unknown w_4, a 40th-order polynomial equation in the unknown h is derived. Of course there are more elegant methods to solve the characteristic equations, e.g., the Sylvester dialytic method of elimination [16, 17] or homotopy continuation [18].

Although the forward displacement analysis presented in this section is easy to follow, the existence of multiple solutions is still a problem of displacement analysis. A simple strategy to determine a closed form solution for the displacement analysis of the robot at hand may be the implementation of a sensor that allows one to determine directly the length h [13]. Then, the unknowns w_1, w_2, w_5 and w_6 are computed recursively, while w_3 is calculated taking into account that from the closure equation $(\boldsymbol{B}_1 - \boldsymbol{C})\cdot(\boldsymbol{B}_1 - \boldsymbol{C}) = e^2$ we have

$$w_3 = \sqrt{e^2 - w_1^2 - (w_2 - h)^2}. \tag{10}$$

Similarly, taking into account that $(\boldsymbol{B}_2 - \boldsymbol{C})\cdot(\boldsymbol{B}_2 - \boldsymbol{C}) = e^2$ one obtains

$$w_4 = \sqrt{e^2 - w_6^2 - (w_5 - h)^2}. \tag{11}$$

Finally, the computation of the unknown coordinates is completed provided that $w_7 = -w_1$ whereas $w_9 = -w_3$ and $w_8 = 2h-w_2$. Once the coordinates of points B_i are determined, the rotation matrix between body p as measured from body 0 is obtained as

$$\mathbf{R} = \begin{bmatrix} \boldsymbol{u}_x & \boldsymbol{u}_y & \boldsymbol{u}_z \end{bmatrix},$$

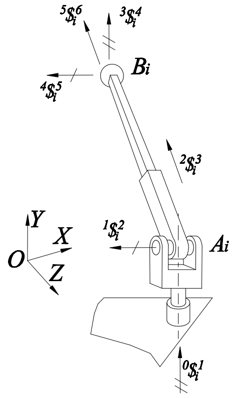

Fig. 2. Modeling of the screws of the parallel manipulator.

where the unit vectors $\boldsymbol{u}$ are given by

$$\boldsymbol{u}_x = \frac{\boldsymbol{B}_1 - \boldsymbol{C}}{e},\ \boldsymbol{u}_y = (\boldsymbol{B}_2 - \boldsymbol{B}_1)\times\left(\frac{\boldsymbol{B}_3 - \boldsymbol{B}_1}{bd}\right),\ \boldsymbol{u}_z = \boldsymbol{u}_x \times \boldsymbol{u}_y .$$

Naturally, the coordinates of any point P attached to the moving platform, expressed in the fixed reference frame, may be obtained as $P = \mathbf{R}P' + C$, where P' are the coordinates of the point in turn expressed in the moving reference frame.

Finally, the inverse displacement analysis consists of finding the generalized coordinates of the robot manipulator given the pose of the moving platform. This analysis is straightforward; see for instance Lu et al. [12].

4. Input-output equations of velocity and acceleration

In this section the velocity and acceleration analyses of the parallel manipulator are addressed by resorting to reciprocal-screw theory. To this end, the modeled of the screws, which are computed taking point C as the reference pole, is illustrated in Fig. 2.

Let $\boldsymbol{\omega}$ be the angular velocity vector of the moving platform as observed from the fixed platform. Furthermore, let $\boldsymbol{v}_C$ be the velocity vector of point C as measured from the fixed reference frame. Then, the velocity state of the moving platform as measured from the fixed platform, the vector $\boldsymbol{V}_C = (\boldsymbol{\omega}, \boldsymbol{v}_C)$ may be expressed in screw form through any of the five limbs of the manipulator as follows

$$\boldsymbol{V}_C = \sum_{i=0}^{5} {}_i\omega_{i+1}^j \ {}^i\$_j^{i+1} \quad j = 1,2,\ldots,5. \tag{12}$$

Therein the joint velocity rates ${}_2\omega_3^j = \dot{q}_j\ (j = 1,2,3,4)$ of the circumferential limbs are selected as the generalized speeds of the manipulator while ${}_2\omega_3^5$ denotes the joint velocity rate associated with the passive prismatic pair of the central limb. Furthermore, a pseudo universal joint connecting the central limb to the fixed platform is included to satisfy an algebraic requirement of expression Eq. (12): The six-dimensional nature of the velocity state. With this assumption in mind it follows that ${}_0\omega_1^5 = {}_1\omega_2^5 = 0$. On the other hand, the modeling of the screws representing the kinematic pairs for each surrounding limb is explained as follows. The screw associated with the revolute joint connecting the *j*th limb to the fixed platform is denoted as ${}^0\$_j^1$ where the primal part of it is normal to the plane of the fixed platform, while ${}^1\$_j^2$ is the screw representing the revolute joint connecting the cylinder to the wrist of the piston. Evidently, the primal part of ${}^1\$_j^2$ lies in the plane of the fixed platform. Dealing with the actuable prismatic joint, the dual part of its representative screw is along the limb. Finally, the spherical joint of the *j*th limb may be simulated by three revolute joints with associated screws ${}^3\$_j^4$, ${}^4\$_j^5$ and ${}^5\$_j^6$ whose primal parts intersect at the center of the spherical joint. Restrictions like parallelism between the primal parts of the screws screws ${}^0\$_j^1$ and ${}^3\$_j^4$ in the same limbs as well as parallelism between the primal parts of the screws screws ${}^1\$_j^2$ and ${}^4\$_j^5$ are added to the spherical joint. Of course, other combinations of screws can be freely selected to simulate the motion of the spherical joint. However, the combination selected in the contribution allows one to simplify considerably the analysis.

Taking into account that in the *j*th surrounding limb of the manipulator the screw ${}^5\$_j^6$ is reciprocal to the remaining screws in the same limb, excluding the screw ${}^2\$_j^3$ representing the actuable prismatic joint, then the systematic application of the Klein form, {;}, of the screw ${}^5\$_j^6$ to both sides of Eq. (12), with the reduction of terms one obtains

$$\left\{ {}^5\$_j^6 ; \boldsymbol{V}_C \right\} = \dot{q}_j \quad j = 1,2,3,4 . \tag{13}$$

Similarly, from the central limb it follows that

$$\left\{ {}^3\$_5^4 ; \boldsymbol{V}_C \right\} = \left\{ {}^4\$^5 ; \boldsymbol{V}_C \right\} = 0. \tag{14}$$

Casting into a matrix-vector form Eqs. (13) and (14) the input-output equation of velocity of the robot manipulator results in

$$\mathbf{A}\boldsymbol{V}_C = \mathbf{Q}_{\mathrm{v}}^T, \tag{15}$$

where $\mathbf{A} = \mathbf{J}^T \Delta$ is named the first-order coefficients matrix of the manipulator in which

$$\mathbf{J} = \begin{bmatrix} {}^5\$_1^6 & {}^5\$_2^6 & {}^5\$_3^6 & {}^5\$_4^6 & {}^3\$_5^4 & {}^4\$_5^5 \end{bmatrix}, \tag{16}$$

is the screw-coordinates Jacobian matrix of the robot manipulator and Δ is an operator of polarity. Meanwhile

$$\mathbf{Q}_{\mathrm{v}} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & 0 & 0 \end{bmatrix} \tag{17}$$

is termed the first-order driver matrix of the parallel manipulator. In what follows, the input-output equation of acceleration of the robot manipulator is obtained based on the strategy outlined for the velocity analysis.

Let $\boldsymbol{\alpha}$ be the angular acceleration vector of the moving platform as measured from the fixed platform. Furthermore, let $\boldsymbol{a}_C$ be the acceleration vector of point *C* as measured from the fixed reference frame. Then, the reduced acceleration state of the moving platform as measured from the fixed platform, the vector $\boldsymbol{A}_C = (\boldsymbol{\alpha}, \boldsymbol{a}_C - \boldsymbol{\omega} \times \boldsymbol{v}_C)$, may be expressed in screw form through any of the five limbs of the manipulator as follows:

$$\boldsymbol{A}_C = \sum_{i=0}^{5} {}_i\alpha_{i+1}^j \; {}^i\$_j^{i+1} + \boldsymbol{L}_j \quad j = 1,2,\ldots,5. \tag{18}$$

Therein ${}_i\alpha_{i+1}^j$ is the joint acceleration rate between consecutive links of the *j*th limb. Meanwhile $\boldsymbol{L}_j$ is the *j*th Lie screw of acceleration which is computed as

$$\boldsymbol{L}_j = \sum_{i=0}^{4} \left[{}_i\omega_{i+1}^j \; {}_i\$_{i+1}^j \quad \sum_{k=i+1}^{5} {}_k\omega_{k+1}^j \; {}_k\$_{k+1}^j \right], \tag{19}$$

where the brackets, [ ], denote the Lie product of the Lie algebra *se(3)* of the Euclidean group *SE(3)*.

Following the trend of the velocity analysis, the input-output equation of acceleration of the parallel manipulator results in

$$\mathbf{A}\boldsymbol{A}_C = \mathbf{Q}_{\mathrm{a}}^T + \mathbf{H}^T, \tag{20}$$

where

$$\mathbf{Q}_{\mathrm{a}} = \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & \ddot{q}_3 & \ddot{q}_4 & 0 & 0 \end{bmatrix},$$

is the second-order driver matrix of the manipulator. Meanwhile, the complementary matrix of acceleration $\mathbf{H}$ is given by

$$\mathbf{H} = \left[\left\{ {}^5\$_1^6 ; \boldsymbol{L}_1 \right\} \left\{ {}^5\$_2^6 ; \boldsymbol{L}_2 \right\} \left\{ {}^5\$_3^6 ; \boldsymbol{L}_3 \right\} \left\{ {}^5\$_4^6 ; \boldsymbol{L}_4 \right\} \left\{ {}^3\$_5^4 ; \boldsymbol{L}_5 \right\} \left\{ {}^4\$_5^5 ; \boldsymbol{L}_5 \right\} \right].$$

5. Numerical example

To show the application of the method, in this section numerical examples covering most of the issues treated in the contribution are provided. Using hereafter SI units (m,rad,s), the dimensions of the platforms of the parallel manipulator are chosen as *a = 1.25 m*, *b = 1.25 m* and *d = 1.0 m*. Meanwhile, the case study comprises the computation of the forward displacement analysis of the parallel manipulator and the temporal behavior of the velocity and acceleration of the center of the moving platform. Dealing with the first part of the case study, given the generalized coordinates $q_1 = 1.85$ *m*, $q_2 = 2.0$ *m*, $q_3 = 1.75$ *m* and $q_4 = 2.1$ *m* it is required to compute all the feasible poses that the moving platform can reach as observed from the fixed platform.

Applying the method introduced in "Displacement analy-

Table 1. The solutions of the forward displacement analysis.

Solution:	*h(m)*
1,2,3:	.1623265480, .8615233449, 1.147295902
4,5,6:	1.175125966, 1.264795049, 1.314496845
21,22,23:	-.1623265480, -.8615233449, -1.147295902
24,25,26:	-1.175125966,-1.264795049, -1.314496845
7,8:	1.746197+.1353975 I, .1453921+0.256352e-1 I
9,10:	.9361554+.217028 I, 1.696551+1.138341 I
11,12:	.1108026+0.8523178e-1 I, 0.7949386e-1 I
13,14,15:	.1414294 I, .1733845 I, .9135315 I
16,17:	-.1108026+0.85231e-1 I, -1.696551+1.1383 I
18,19:	-.9361554+.217028 I,-.1453921+0.256352e-1 I
20:	-1.746197306+.1353975548 I
27,28:	-1.746197-.1353975 I, -.1453921-0.256352e-1 I
29,30:	-.9361554-.217028 I, -1.696551-1.138341 I
31,32,33:	-.1108-0.8523e-1 I, -0.794e-1*I,-.14142 I
34,35,36:	-.1733845 I,-.91353 I, .110802-0.8523e-1 I
37,38:	1.69655-1.13834 I, .9361554234-.2170280014 I
39,40:	.1453921-0.256352e-1 I, 1.74619-.1353975548 I

Table 2. Available coordinates of points B_i.

Solution: B_1,B_2,B_3,B_4	
Solution 3:	(0.0191,1.225,-0.636), (-0.910,1.057,0.182), (-0.019,1.068,0.636), (0.910,1.236,-0.182)
Solution 6:	(0.105,1.382,0.447), (-1.027,1.236,-0.060), (-0.105,1.245,-0.447), (1.027,1.392,0.060)
Solution 23:	(0.110,-1.225,-0.788), (0.746,-1.057,0.274), (-0.110,-1.068,0.788), (-0.746,-1.236,-0.274)
Solution 26:	(0.275,-1.382,0.748), (-0.789,-1.236,0.109), (-0.275,-1.246,-0.748), (0.789,-1.392,,-0.109)

sis" section, one obtains

$$.03705625+.32h^6-(.9732+.8w4-.8w3)h^4-(.748451375-w4^2-.8315w4-w3^2+1.6015w3)h^2 = 0,$$
$$.00015625+.32h^6-(.9732-.8w4+.8w3)h^4-(.185951375-w4^2-w3^2-1.6015w3+.8315w4)h^2 = 0,$$
$$.0324+.64h^6-1.3304h^4-(1.871107750-4w3^2)h^2 = 0.$$

Squaring selectively the characteristic equations, a 40th-order polynomial equation in the unknown h is promptly generated. The solutions of it are provided in Table 1.

Taking into account that spurious and complex solutions must be disregarded from the analysis, the available poses of the parallel manipulator are related only with solutions 3, 6, 23 and 26 of Table 1. The corresponding coordinates of the spherical joints are provided in Table 2.

In what follows the temporal behavior of the angular and linear components of the velocity and acceleration of the center of the moving platform is carried-out by means of the theory of screws. Conveniently, at the beginning of the motion the robot is in a symmetric posture where the coordinates of the vertices of the fixed platform are given by $A_1 = (1.25,0,0)$, $A_2 = (0,0,-1.25)$, $A_3 = (-1.25,0,0)$ and $A_4 = (0,0,1.25)$. Meanwhile, the surrounding limbs of the parallel manipulator assume to have the same length $q = 2.0\,m$. Furthermore, the coordinates of points B_i are given by $B_1 = (0.795,1.945,0.088)$, $B_2 = (-0.088,1.945,-0.795)$, $B_3 = (-0.795,1.945,-0.088)$ and $B_4 = (0.088,1.945,0.795)$. On the other hand, upon the reference configuration of the robot, the generalized coordinates q are commanded to follow periodical functions given by $q_1 = 0.1sin(t)cos(t), q_2 = 0.25sin(t)cos(t), q_3 = 0.125sin(t)cos(t)$ and $q_4 = -0.15sin(t)cos(t)$ where the time t is in the interval $0 < t < 2\pi$.

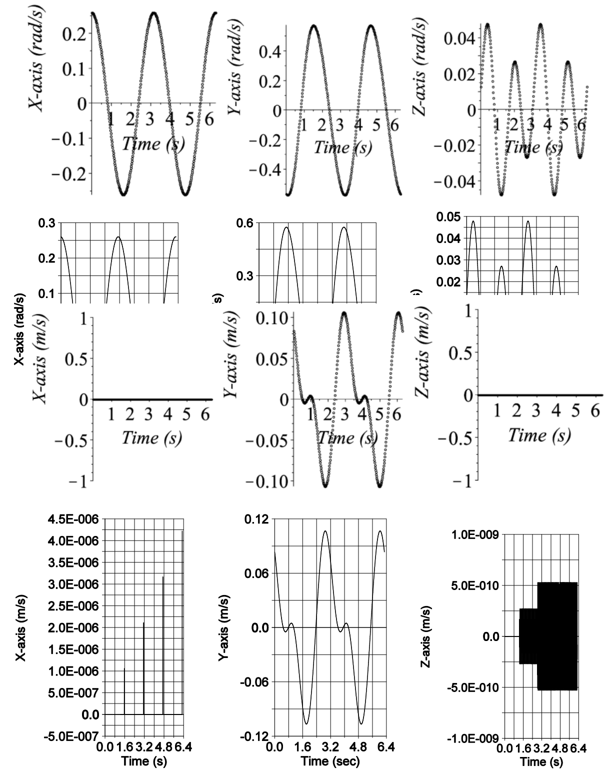

Fig. 3. Time history of the velocity of the center of the moving platform.

With the aforementioned data, the temporal behavior of the velocity and acceleration of the center of the moving platform is provided in Figs. 3 and 4, respectively. Furthermore, the numerical results obtained by means of the theory of screws are verified with the aid of commercially available software like ADAMS©. For a rapid comparison between both methods, the corresponding plots obtained with ADAMS© are placed below the simulations generated via screw theory.

Note that, as was expected, the linear velocity and acceleration of the center of the moving platform concerned with the X and Y axes vanish. Furthermore, the numerical results obtained by means of the theory of screws are in excellent agreement with those generated with ADAMS©.

6. Conclusions

Owing to relevant applications like laparoscopic surgery, ship's heave and swing motion simulation, robots for generating Schöenflies motion, hip joint simulator, pick-and-place robot, aeronautical devices and so far of four-degrees-of-

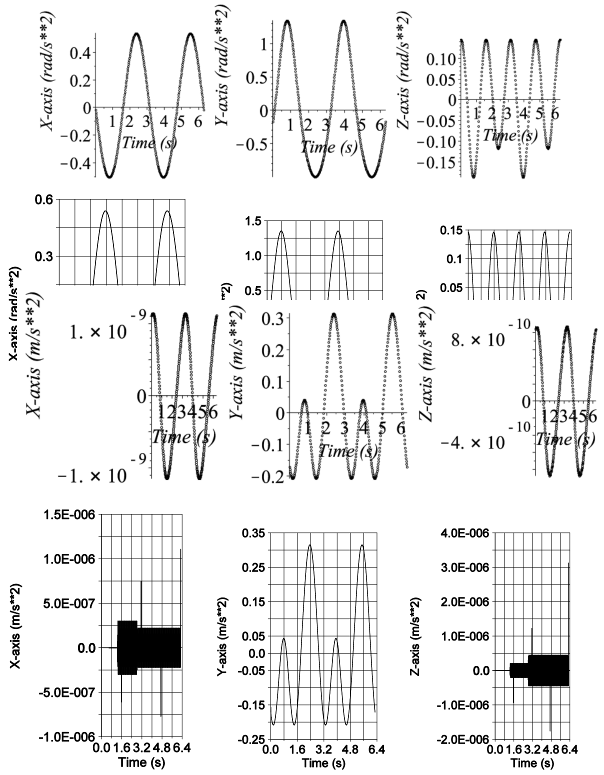

Fig. 4. Time history of the acceleration of the center of the moving platform.

freedom parallel manipulators, in this contribution a simple approach to solve the kinematics of the 4-UPS/PS *(3R1T*) parallel manipulator is presented. Closure equations to solve the displacement analysis of the parallel manipulator are easily formulated upon linear combinations of the unknown coordinates of three points embedded in the moving platform, namely, the control points of the manipulator which allow one to obtain nearly closed-form solutions concerned with the forward displacement analysis, a challenging task for most parallel manipulators. As far as the authors are aware, this idea has not been considered in previous works for the robot under study, e.g. this method does not require one to resort to standard mathematical procedures like the Sylvester dialytic method of elimination or homotopy continuation. Then, the input-output equations of velocity and acceleration of the robot manipulator are systematically established by taking advantage of the properties of reciprocal screws. Numerical examples are provided to show the application of the method. Finally, comments like ``*Unfortunately, screw theory is usually explained following descriptive definitions rather than short axiomatic lines of reasoning*", ``*Screw theory allows simple geometrical interpretation, but it is restricted to speed and infinitesimal displacement analysis*", can be found in the specialized literature. The contribution shows that the theory of screws is a viable and trusted mathematical resource to approach not only the velocity analysis but also the acceleration analysis of spatial kinematic chains.

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