PERFORMANCE ANALYSIS OF THE FAMILY OF CONJUGATE GRADIENT ITERATIVE METHODS WITH NON-POLYNOMIAL SPLINE SCHEME FOR SOLVING SECOND- AND FOURTH-ORDER TWO-POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT

A numerical solution involving two-point boundary value problems has vast contributions especially to formulate problems mathematically in fields such as science, engineering, and economics. In response to that, this study was conducted to solve for the second- and fourth-order two-point boundary value problems (BVPs) by using cubic and quartic non-polynomial spline discretization schemes for full-, half- and quarter-sweep cases. The derivation process based on the cubic and quartic non-polynomial spline functions were implemented to generate the full-, half- and quarter-sweep cases non-polynomial spline approximation equations. After that, the non-polynomial spline approximation equations were used to generate the corresponding systems of linear equations in a matrix form. Since the systems of linear equations have large and sparse coefficient matrices, therefore the linear systems were solved by using the family of Conjugate Gradient (CG) iterative method. In order to conduct the performances comparative analysis of the CG iterative method, there are two other iterative methods were considered which are Gauss-Seidel (GS) and Successive-Over-Relaxation (SOR) along with the full-, half- and quarter-sweep concepts. Furthermore, the numerical experiments were demonstrated by solving three examples of second- and fourth-order two-point BVPs in order to investigate the performance analysis in terms of the number of iterations, execution time and maximum absolute error. Based on the numerical results obtained from the implementation of the three iteration families together with the cubic and quartic non-polynomial spline schemes, the performance analysis of the CG iterative method was found to be superior to the GS and SOR iteration families in terms of the number of iteration, execution time and maximum absolute error when solving the two-point BVPs. Hence, it can be stated that the CG iteration family is more efficient and accurate than the GS and SOR iteration families when solving the second-order two-point BVPs based on the cubic and quartic non-polynomial spline schemes. However, for the fourth-order two-point BVPs, the numerical results have shown that the implementation of the CG iteration family over the reduced system of second-order two-point BVPs failed to satisfy the convergence iteration criteria. As a result, the SOR iteration family is superior to GS iteration family in terms of the number of iteration, execution time and maximum absolute error.

ABSTRAK

ANALISIS PRESTASI FAMILI KAEDAH LELARAN KECERUNAN KONJUGAT DENGAN SKEMA SPLIN TAK POLINOMIAL TERHADAP MASALAH NILAI SEMPADAN DUA-TITIK PADA PERINGKAT KEDUA DAN KEEMPAT

Penyelesaian berangka yang melibatkan masalah nilai sempadan dua-titik mempunyai kepentingan yang meluas untuk merumuskan permasalahan tersebut secara matematik dalam pelbagai bidang seperti sains, kejuruteraan dan ekonomi. Sehubungan dengan itu, kajian ini dijalankan untuk menyelesaikan masalah nilai sempadan dua-titik dengan menggunakan skema pendiskretan splin tak polinomial kubik dan kuartik yang merangkumi kes sapuan penuh, separuh dan suku. Proses pendiskretan terhadap fungsi splin tak polinomial kubik dan kuartik telah dilaksanakan untuk menerbitkan persamaan penghampiran splin bagi ketiga-tiga kes sapuan penuh, separuh dan suku. Seterusnya, persamaan penghampiran splin tersebut digunakan untuk menjana sistem persamaan linear yang sepadanan dalam bentuk matrik. Memandangkan sistem persamaan linear tersebut mempunyai pekali matriks yang berskala besar, maka sistem persamaan linear tersebut diselesaikan dengan menggunakan famili kaedah lelaran Kecerunan Konjugat (KK). Bagi menjalankan analisis perbandingan prestasi terhadap famili kaedah lelaran KK, terdapat dua famili kaedah lelaran lain yang turut dijalankan iaitu famili kaedah lelaran Gauss-Seidel (GS) dan Pengenduran Berlebihan Berturut-turut (PBB) bersama dengan konsep sapuan penuh, separuh dan suku. Selanjutnya, ujian berangka telah didemonstrasikan dengan menyelesaikan tiga permasalahan nilai sempadan dua-titik bagi setiap peringkat kedua dan keempat untuk mengkaji analisis prestasi dari aspek bilangan lelaran, masa lelaran dan ralat mutlak maksimum. Berdasarkan keputusan ujian berangka ke atas permasalahan tersebut dengan menggunakan famili kaedah lelaran GS, PBB dan KK, famili kaedah lelaran KK telah menunjukkan prestasi yang lebih baik dari aspek bilangan lelaran, masa lelaran dan ralat mutlak maksimum berbanding dengan prestasi famili kaedah lelaran GS dan PBB. Justeru itu, dapat dinyatakan bahawa famili kaedah lelaran KK adalah lebih efisien dan jitu berbanding dengan famili kaedah lelaran GS dan SOR dalam menyelesaikan masalah nilai sempadan dua-titik pada peringkat kedua berdasarkan persamaan penghampiran splin tak polinomial kubik dan kuartik. Walaubagaimanapun, dalam kes masalah nilai sempadan dua-titik peringkat keempat, keputusan uji berangka menunjukkan bahawa famili kaedah lelaran KK bersama dengan pendekatan splin tak polinomial kubik dan kuartik telah gagal memenuhi kriteria penumpuan lelaran dalam menyelesaikan masalah nilai sempadan dua-titik peringkat kedua terturun. Sehubungan dengan itu, didapati bahawa famili kaedah lelaran PBB adalah lebih baik berbanding dengan famili kaedah lelaran GS dari aspek bilangan lelaran, masa lelaran dan ralat mutlak maksimum dalam menyelesaikan masalah nilai sempadan dua-titik peringkat kedua terturun.

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LIST OF ABBREVIATIONS

ADM	Adomian Decomposition Method
AGE	Alternating Group Explicit
AM	Arithmetic Mean
AOR	Accelerated Over-Relaxation
BGS	Backward Gauss-Seidel
BVPS	Boundary Value Problems
CG	Conjugate Gradient
CGNR	Conjugate Gradient Normal Residual
EDG	Explicit Decoupled Group
EG	Explicit Group
EGSOR	Explicit Group Successive Over-Relaxation
EGMSOR	Explicit Group Modified Successive Over-Relaxation
FCGNR	Full-Sweep Conjugate Gradient Normal Residual
FGS	Forward Gauss-Seidel
FSAOR	Full-Sweep Accelerated Over-Relaxation
FSAM	Full-Sweep Arithmetic Mean
FSGM	Full-Sweep Geometric Mean
FSGS	Full-Sweep Gauss-Seidel
FSSOR	Full-Sweep Successive Over-Relaxation
GMRES	Generalized Minimal Residual
GS	Gauss-Seidel
HCGNR	Half-Sweep Conjugate Gradient Normal Residual
HSAGE	Half-Sweep Alternating Group Explicit
HSAM	Half-Sweep Arithmetic Mean
HSAOR	Half-Sweep Accelerated Over-Relaxation
HSCG	Half-Sweep Conjugate Gradient
HSGM	Half-Sweep Geometric Mean
HSGS	Half-Sweep Gauss-Seidel
HSIADE	Half-Sweep Iterative Alternating Decomposition Explicit

HSSOR	Half-Sweep Successive Over-Relaxation
HSSOR9L	Half-Sweep Successive Over-Relaxation via Nine-Point Laplacian
HSMSOR	Half-Sweep Modified Successive Over-Relaxation
IDE	Integro-Differential Equation
IVP	Initial Value Problem
MEDG	Modified Explicit Decoupled Group
MEG	Modified Explicit Group
MINERS	Minimal Residual method
MSOR	Modified Successive Over-Relaxation
Newton-AGE	Newton Alternating Group Explicit
Newton-SOR	Newton Successive Over-Relaxation
ODES	Ordinary Differential Equations
PTI	Precise Time Integration
QSAGE	Quarter Sweep Alternating Group Explicit
QSAOR	Quarter-Sweep Accelerated Over-Relaxation
QSAM	Quarter-Sweep Arithmetic Mean
QSCG	Quarter-Sweep Conjugate Gradient
QSCGNR	Quarter-Sweep Conjugate Gradient Normal Residual
QSCN	Quarter-Sweep Crank-Nicolson
QSGM	Quarter-Sweep Geometric Mean
QSGS	Quarter-Sweep Gauss-Seidel
QSSOR	Quarter Sweep Successive Over-Relaxation
RKM	Reproducing Kernel Method
SOR	Successive Over-Relaxation

LIST OF SYMBOLS

σ_i	- Coefficient matrix for second-order two-point BVPs based on cubic non-polynomial spline
ϕ_i	- Coefficient matrix for second-order two-point BVPs based on cubic non-polynomial spline
δ_i	- Coefficient matrix for second-order two-point BVPs based on cubic non-polynomial spline
$\sigma_i^{\#}$	- Coefficient matrix for second-order two-point BVPs based on quartic non-polynomial spline
$\phi_i^{\#}$	- Coefficient matrix for second-order two-point BVPs based on quartic non-polynomial spline
$\delta_i^{\#}$	- Coefficient matrix for second-order two-point BVPs based on quartic non-polynomial spline
σ_i^{*}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
ϕ_i^{*}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
δ_i^{*}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
σ_i^{**}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
ϕ_i^{**}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
δ_i^{**}	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on cubic non-polynomial spline
$\sigma_i^{\#*}$	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
$\phi_i^{\#*}$	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
$\delta_i^{\#*}$	- Coefficient matrix for first linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
$\sigma_i^{\#**}$	- Coefficient matrix for second linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
$\phi_i^{\#**}$	- Coefficient matrix for second linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
$\delta_i^{\#**}$	- Coefficient matrix for second linear system of reduced fourth-order two-point BVPs based on quartic non-polynomial spline
A_i	- Coefficient matrix for second-order two-point BVPs based on cubic non-polynomial spline

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