Hankel Determinant $H_2(3)$ for Certain Subclasses of Univalent Functions

ABSTRACT

Let S to be the class of functions which are analytic, normalized and univalent in the unit disk $U = \{z : |z| < 1\}$. The main subclasses of S are starlike functions, convex functions, close-to-convex functions, quasiconvex functions, starlike functions with respect to (w.r.t.) symmetric points and convex functions w.r.t. symmetric points which are denoted by S^*, K, C, C^*, S^*_S, and K_S respectively. In recent past, a lot of mathematicians studied about Hankel determinant for numerous classes of functions contained in S. The qth Hankel determinant for $q \geq 1$ and $n \geq 0$ is defined by $H_q(n)$. $H_2(1) = a_3 - a_2^2$ is greatly familiar so called Fekete-Szego functional. It has been discussed ¨ since 1930's. Mathematicians still have lots of interest to this, especially in an altered version of $a_3 - \mu a_2^2$. Indeed, there are many papers explore the determinants $H_2(2)$ and $H_3(1)$. From the explicit form of the functional $H_3(1)$, it holds $H_2(k)$ provided k from 1-3. Exceptionally, one of the determinant that is $H_2(3) = a_3a_5 - a_4^2$ has not been discussed in many times yet. In this article, we deal with this Hankel determinant $H_2(3) = a_3a_5 - a_4^2$. From this determinant, it consists of coefficients of function f which belongs to the classes S^*_S and K_S so we may find the bounds of $|H_2(3)|$ for these classes. Likewise, we got the sharp results for S^*_S and K_S for which $a_2 = 0$ are obtained.