Approximation solution of the fractional parabolic partial differential equation by the half-sweep and preconditioned relaxation

ABSTRACT

In this study, the numerical solution of a space-fractional parabolic partial differential equation was considered. The investigation of the solution was made by focusing on the space-fractional diffusion equation (SFDE) problem. Note that the symmetry of an efficient approximation to the SFDE based on a numerical method is related to the compatibility of a discretization scheme and a linear system solver. The application of the one-dimensional, linear, unconditionally stable, and implicit finite difference approximation to SFDE was studied. The general differential equation of the SFDE was discretized using the space-fractional derivative of Caputo with a half-sweep finite difference scheme. The implicit approximation to the SFDE was formulated, and the formation of a linear system with a coefficient matrix, which was large and sparse, is shown. The construction of a general preconditioned system of equation is also presented. This study's contribution is the introduction of a half-sweep preconditioned successive over relaxation (HSPSOR) method for the solution of the SFDE-based system of equation. This work extended the use of the HSPSOR as an efficient numerical method for the time-fractional diffusion equation, which has been presented in the 5th North American International Conference on industrial engineering and operations management in Detroit, Michigan, USA, 10–14 August 2020. The current work proposed several SFDE examples to validate the performance of the HSPSOR iterative method in solving the fractional diffusion equation. The outcome of the numerical investigation illustrated the competence of the HSPSOR to solve the SFDE and proved that the HSPSOR is superior to the standard approximation, which is the full-sweep preconditioned SOR (FSPSOR), in terms of computational complexity.