Quarter-sweep preconditioned relaxation method, algorithm and efficiency analysis for fractional mathematical equation

ABSTRACT

Research into the recent developments for solving fractional mathematical equations requires accurate and efficient numerical methods. Although many numerical methods based on Caputo's fractional derivative have been proposed to solve fractional mathematical equations, the efficiency of obtaining solutions using these methods when dealing with a large matrix requires further study. The matrix size influences the accuracy of the solution. Therefore, this paper proposes a quarter-sweep finite difference scheme with a preconditioned relaxation-based approximation to efficiently solve a large matrix, which is based on the establishment of a linear system for a fractional mathematical equation. The paper presents the formulation of the quarter-sweep finite difference scheme that is used to approximate the selected fractional mathematical equation. Then, the derivation of a preconditioned relaxation method based on a quarter-sweep scheme is discussed. The design of a C++ algorithm of the proposed quarter-sweep preconditioned relaxation method is shown and, finally, efficiency analysis comparing the proposed method with several tested methods is presented. The contributions of this paper are the presentation of a new preconditioned matrix to restructure the developed linear system, and the derivation of an efficient preconditioned relaxation iterative method for solving a fractional mathematical equation. By simulating the solutions of time-fractional diffusion problems with the proposed numerical method, the study found that computing solutions using the quarter-sweep preconditioned relaxation method is more efficient than using the tested methods. The proposed numerical method is able to solve the selected problems with fewer iterations and a faster execution time than the tested existing methods. The efficiency of the methods was evaluated using different matrix sizes. Thus, the combination of a quarter-sweep finite difference method, Caputo's time-fractional derivative, and the preconditioned successive over-relaxation method showed good potential for solving different types of fractional mathematical equations, and provides a future direction for this field of research.