SECRETS BEHIND THE NIM GAME

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ABSTRACT

This dissertation discusses the nim game in detail. Nim game is a traditional two-person game where players alternatively take turns in removing counters (objects) from the piles of counters. The purposes of this study are to identify different ways in winning the nim game mathematically, to modify the standard nim game and also use game trees in winning the nim game. The number of counters use to play this game in this research vary from one to thirty and the number of piles are within the range from one to four. In determining the winning ways for the nim game, a few methods are used, namely the binary concept, *XOR* (bitwise-exclusive-or) and game trees. Binary concept includes changing decimal numbers to binary numbers and the other way round. *XOR* is a concept where binary numbers are added without carry. This concept is used to determine the winning position of the game. Modifications are done to the standard nim game in order to make it a game of chance. In this dissertation, the nim game is firstly modified by blocking certain moves and secondly by modifying the coin game. These modifications allow equal chances of both the players to win the game. Finally, game trees are illustrations formed to represent the nim game with small amount of piles and counters. The winner of the game is the last player who removes the counter except for one variation discussed which is the misère nim. In conclusion, all the objectives of this dissertation have been achieved. These include obtaining different winning methods mathematically for the nim game, successfully modifying the nim game to obtain a game of chance and finally winning the nim game through game trees.

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RAHSIA DI SEBALIK PERMAINAN NIM

ABSTRAK

Disertasi ini membincangkan permainan nim secara mendalam. Permainan nim adalah suatu permainan tradisional di antara dua orang di mana pemain-pemain mengambil sebilangan objek (batu, guli dan sebagainya) daripada jumlah objek yang dilonggokkan. Tujuan kajian ini dijalankan adalah untuk mengenalpasti kaedah yang digunakan dalam memenangi permainan nim dengan menggunakan konsep matematik, mengubahsuai permainan nim yang sedia ada dan menggunakan gambar rajah pokok bagi memenangi permainan nim. Bilangan objek yang digunakan di dalam kajian ini adalah di antara satu hingga tiga puluh dan bilangan longgokkan objek-objek ini adalah di antara julat satu hingga empat. Beberapa konsep diperkenalkan bagi memenangi permainan nim iaitu, konsep binari, *XOR* (bitwise-exclusive-or) dan gambar rajah pokok. Konsep binari yang digunakan ialah penukaran nombor asas sepuluh kepada nombor asas dua dan juga sebaliknya. *XOR* pula adalah hasil tambah nombor-nombor binari tanpa baki. Konsep ini digunakan bagi menjanakan kaedah bagi memenangi permainan nim secara matematik. Pengubahsuaian dilakukan ke atas permainan nim untuk menukarkan sifatnya yang 'sentiasa boleh menang' kepada permainan yang boleh dimenangi secara adil dan ia dilakukan melalui dua cara. Yang pertama adalah melalui kaedah memblok sesetengah nombor dan kedua melalui pengubahsuaian 'coin game'. Pengubahsuaian ini dilakukan untuk memberikan peluang yang sama bagi kedua-dua pemain memenangi permainan ini. Gambarajah pokok digunakan bagi permainan nim dengan jumlah objek dan longgokkan yang kecil. Pemenang bagi permainan ini adalah pemain yang mengambil objek yang terakhir daripada longgokkan kecuali satu variasi permainan nim yang dibincangkan iaitu misère nim yang pemenangnya adalah pemain yang tidak mengambil objek terakhir. Sebagai kesimpulan, kesemua objektif bagi disertasi ini dicapai iaitu, kaedah matematik memenangi permainan nim, mengubahsuai permainan nim kepada bentuk yang lebih menarik dan memenangi permainan nim menggunakan gambar rajah pokok.

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LIST OF SYMBOLS

$>$	greater than
$<$	lesser than
$\leq$	lesser than or equals to
$\geq$	greater than or equals to
$\in$	member / object to
XOR	bitwise-exclusive-or
$f(n)$	function to *n*

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Whether or not Alain Resnais' movie, L'année der nière à Marienbad (Last Year in Marienbad) hit the box office in the year 1962, it sure did hit something into mathematicians of that time. In that movie, a game with matches was played (Final Answers, 2005). The game which was played in a different version managed to capture the hearts of mathematicians who till today are doing researches about the game. The name of the game was nim and through that movie the nim game gained its fame.

The name is probably derived from German '*nimm*' which means "take!" Some people have noted that turning the word NIM results in WIN, but this is probably just a coincidence (Nim, 2005). According to Gough (1999), a game needs to have two or more players, who take turns each competing to achieve a winning situation of some kind, each able to exercise some choice about how to move any time through the playing. The nim game does fulfill all the descriptions of a game set by Gough.

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The following discussion shows how the nim game works. There are j heaps, each with $i_1, i_2, ..., i_j$ pebbles in each heap. The first player chooses a pile and removes one up to all the pebbles in that particular pile. According to Brauldi, players may choose only one pile at a time to remove objects from. This process continues until the last pebble is removed – and the player who does so is the winner. This is the standard way of playing the nim game. Other than pebbles, other objects can be used to play the nim game. These include matches, buttons, coins and more. These objects are known as counters. Some do not use objects as they prefer playing the nim game manually, which is to draw lines and cancel off the lines as a symbol that the counter has been removed.

These are some ways how the objects for a standard nim game can be arranged:

Figure 1.1 The classical arrangements of a nim game.

Figure 1.2 The nim game with one pile.

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Figure 1.3 Nim game with a few piles.

OOOOO	→	**OOOOO**
OOOOOO		**OOØØØØ**

O — the original number of counters that have been drawn

Ø — the counters that has been cancelled off as a sign of removing it from the pile

Figure 1.4 Playing nim the manual way.

The winner of a nim game can be pre-determined. Due to this extra feature, the nim game is known as a game of no chance. This is because, the nim game carries a mathematical concept in winnning. Therefore, if either one of the player knows this concept, he or she will definitely win at each game causing absolutely no chance for the opponent to win. We can say that the nim game is a mathematical game of strategy as it involves knowledge of mathematical concepts to win the game. The mathematical concept used is the binary number system. Knowing the application of this concept ensures a player the winning position as long as certain conditions are fulfilled.

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1.2 NIM GAME AS A GAME OF CHANCE

Nim game is a game of mathematical concept and a game of no chance. In a game of no chance, a player who understands the underlying concept of nim game can surely win the game if he or she is the first player or he or she is playing against someone who does not know the concept. Therefore, mathematicians have come up with ways to modify the nim game of no chance into a nim game of chance. After these modifications, winning the nim game purely depends on the player and not the mathematical concept. These are some of the modifications:

a. Blocking Nim

The basic rules for the modified version of this nim game is the same. In this game of blocking nim, two players take alternate turns in playing. The name of this modified nim is given based on the procedures that we do in the game. This blocking nim game is not like any other nim game because it does not require counters but it is played with paper and pencil. This game which has been modified is played only with one pile.

This is how the game is played. A number will be given at the beginning of the game. That number is reduced through subtraction. In the process of reducing the number, the opponent does not allow it by blocking the numbers. This game ends when either one of the player is incapable of moving or the total number given in the beginning becomes negative. Other than that, there are also a few more conditions and

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assumptions made in this game that is discussed in this dissertation. The winner of this game is the player who manages to subtract the last number. Through this brief explanation of the blocking nim, we can see that it is not a game of no chance because there are blocks and these blocks are not always the same.

b. Coin Game

This modified version on the nim game comes from the original version of the coin game. Unlike the blocking nim game, this game is played with counters and are put in piles. As usual, two players alternately take turns in removing counters from the piles. What differentiates this modified nim game from the standard one is that in this game, the number of counters to be removed at each turn is determined by both the players.

This is how the modified coin game is played. Two pairs of integers are chosen where in each pair there are two different numbers. The first pair belongs to the present player while the second pair belongs to his opponent. The present player chooses a pile of counters and tells the opponent. Then, both the players write down a number which is in between the pairs of integers assigned to them. This is a secretive act. When they have written it down, they reveal the numbers to each other. Then, the numbers are added together. If the number is more than the number of counters in the pile, the present player removes the whole pile. If the number of counters are more than the addition of both the integers, the number of counters removed are exactly the addition of both the integers. When it comes to the next round, the current opponent will be the

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present player so he will have the first pair of integers and his opponent the second pair of integers.

The details of this modified coin game are further discussed in this dissertation. Through the brief introduction of the game, we can say that this modified version of coin game is really a game of chance. This is because both players decide the amount of counters to be removed together.

1.3 BINARY CONCEPT IN NIM GAME

We use the decimal number system as we have ten fingers. The main advantage of this system is that it uses place-value notation (or positional notation) as the value of each figure depends on its position (Number Systems, 2002). However, there are other number systems too. The number system that will be used to analyze the nim game is the binary system. Binary numbers are numbers which are written in base 2. Therefore, the symbols involved in writing binary numbers are 0 and 1. The theoretical mechanism which is used to make computers work is also based on the binary number system (Number Systems, 2002).

In the year 1902, C.L. Bouton from Harvard University discovered the way where standard nim game can be won using a mathematical concept. This concept involves the application of binary numbers. Winning the nim game with the binary concept has its own strategy. The strategy used to win the game is to make certain

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calculations in base 2. The game is used as a way of introducing some initial concepts about number systems (Number Systems. 2002).

The ways binary numbers are used in winning the nim game will be further discussed in this dissertation.

1.4 GAME TREES IN NIM GAME

Other than the application of binary numbers in winning the nim game, game trees are another alternative to winning the nim game. Game trees can be used for nim games that involve a small number of counters and piles. This method is usually taught to children who have not learnt binary concepts but do know the nim game. Figure 1.5 portrays an example of a typical game tree.

Now, with the game tree, we can analyze the possible moves that can be made. This game tree represents a nim game of three piles and each pile has one, two and two counters respectively which can be written as (1, 2, 2). The first player has three options in removing his counters. He can remove one or two counters in his first move which will leave him with either (1, 2), (2, 2) or (1, 1, 2) counters. The pile positions (1, 2) and (1, 1, 2) are at losing position because the opponent has the power to remove counters where at least one of the possible arrangements after the counters are being removed can cause the first player to lose.

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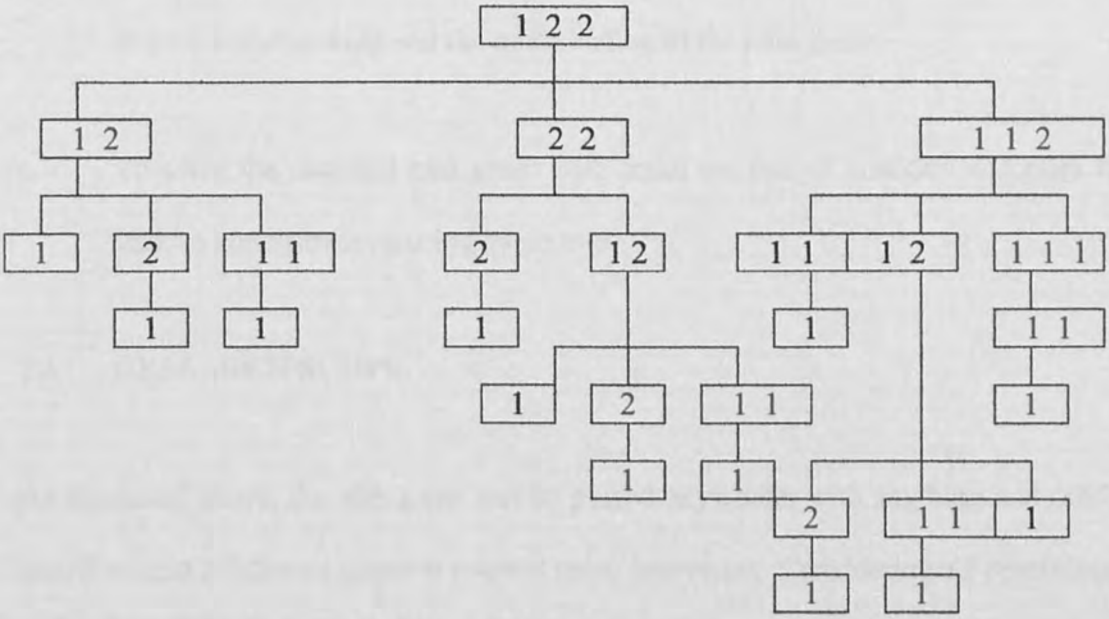
From the game tree from Figure 1.5, we can see the usage of two different colours. The numbers in black represent the winning position for the first player whereas the numbers in red are the player's losing position. The benefits attained from playing with a game tree is that a particular player has the chance of having a view of the whole game even before playing the game and as he's playing the game he enables himself to make careful moves so that he's always at the winning position



█ - losing position for the first player

█ - winning position of the first player.

Figure 1.5 Game tree in solving the nim game.

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1.5 RESEARCH OBJECTIVES

The objectives of this research are:

a. To compare the differences between the winning strategy of the standard nim game and the misère nim game by applying the binary concept.

b. To modify the standard nim game from a game of no chance into a game of chance with blocking and the modification of the coin game

c. To solve the standard nim game with small number of counters and piles for various configurations using game trees.

1.6 RESEARCH SCOPE

As discussed above, the nim game can be played anywhere, with anything and can be modified into a different game to make it more interesting. Considering all possibilities for this dissertation would be rather impossible. Therefore, we have set a scope of what this dissertation will be about. This dissertation will mainly revolve around the three objectives mentioned above.

The nim game in this research will be played based on the requirements set by each objective which has been mentioned above. There are no particular settings of the arrangements of the nim game because all types of arrangements are being used.

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The number of piles is within the range of one to four. The number of counters per pile will be within the range one to thirty. These particular arrangements are chosen because the number of counters and the number of piles do not play a role in determining the winner or having a winning position. By having smaller number of counters and piles, the concept of the game can be understood more deeply and discussed in more complete manner. Colours that are being used in this dissertation do have meaning which is connected to the research. Colours in this context are mainly used in game trees where the winning and losing points are showed.

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CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, we will be discussing about the origin and development of the nim game, the details concerning a game tree and the development of the binary system. This chapter also gives an outlook of different types of nim game and how it has advanced over the years of research.

2.2 THE BIRTH OF NIM GAME

According to Martin Gardner, nim game is one of the oldest two person game known till today (Bogomolny, 2001). The nim game is believed to have originated from China. In the 1500's, a similar game was mentioned by an Italian Mathematician by the name Luca Pacioli (The Game of Nim, 2005). In the year 1902, C.L. Bouton, an associate professor of Mathematics at Harvard University first analyzed this game. He found the solution for this game and that is now viewed as the birth of combinatorial game theory (Combinatorial Game Theory Background, 2005). The name nim was given by C.L. Bouton. This name is believed to have originated from a German verb

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