**Square Integer Matrix with a Single Non-Integer Entry in Its Inverse**

**ABSTRACT**

Matrix inversion is one of the most significant operations on a matrix. For any non-singular matrix $A \in Z\ n\times n$ , the inverse of this matrix may contain countless numbers of non-integer entries. These entries could be endless floating-point numbers. Storing, transmitting, or operating such an inverse could be cumbersome, especially when the size n is large. The only square integer matrix that is guaranteed to have an integer matrix as its inverse is a unimodular matrix $U \in Z\ n\times n$ . With the property that $\det(U) = \pm1$, then $U-1 \in Z\ n\times n$ is guaranteed such that $UU-1 = I$, where $I \in Z\ n\times n$ is an identity matrix. In this paper, we propose a new integer matrix $G\tilde{}\ \in Z\ n\times n$ , which is referred to as an almost-unimodular matrix. With $\det(G\tilde{})\ 6= \pm1$, the inverse of this matrix, $G\tilde{}\ -1 \in Rn\times n$ , is proven to consist of only a single non-integer entry. The almost-unimodular matrix could be useful in various areas, such as lattice-based cryptography, computer graphics, lattice-based computational problems, or any area where the inversion of a large integer matrix is necessary, especially when the determinant of the matrix is required not to equal ±1. Therefore, the almost-unimodular matrix could be an alternative to the unimodular matrix.