Solving One-Dimensional Porous Medium Equation Using Unconditionally Stable Half-Sweep Finite Difference and SOR Method

ABSTRACT

A porous medium equation is a nonlinear parabolic partial differential equation that presents many physical occurrences. The solutions of the porous medium equation are important to facilitate the investigation on nonlinear processes involving fluid flow, heat transfer, diffusion of gas-particles or population dynamics. As part of the development of a family of efficient iterative methods to solve the porous medium equation, the Half-Sweep technique has been adopted. Prior works in the existing literature on the application of Half-Sweep to successfully approximate the solutions of several types of mathematical problems are the underlying motivation of this research. This work aims to solve the one-dimensional porous medium equation efficiently by incorporating the Half-Sweep technique in the formulation of an unconditionally-stable implicit finite difference scheme. The noticeable unique property of Half-Sweep is its ability to secure a low computational complexity in computing numerical solutions. This work involves the application of the Half-Sweep finite difference scheme on the general porous medium equation, until the formulation of a nonlinear approximation function. The Newton method is used to linearize the formulated Half-Sweep finite difference approximation, so that the linear system in the form of a matrix can be constructed. Next, the Successive Over Relaxation method with a single parameter was applied to efficiently solve the generated linear system per time step. Next, to evaluate the efficiency of the developed method, deemed as the Half-Sweep Newton Successive Over Relaxation (HSNSOR) method, the criteria such as the number of iterations, the program execution time and the magnitude of absolute errors were investigated. According to the numerical results, the numerical solutions obtained by the HSNSOR are as accurate as those of the Half-Sweep Newton Gauss-Seidel (HSNGS), which is under the same family of Half-Sweep iterations, and the benchmark, Newton-Gauss-Seidel (NGS) method. The improvement in the numerical results produced by the HSNSOR is significant, and requires a lesser number of iterations and a shorter program execution time, as compared to the HSNGS and NGS methods.