NATURAL CONVECTION NEAR A STAGNATION POINT ABOUT A CIRCULAR CYLINDER

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Abstrak

Tesis ini membincangkan masalah perolakan tabii pada satu silinder membulat mengufuk yang tak terhingga panjangnya. Suhu pada permukaan silinder berayun dengan frekuensi w *dan amplitud* bT'_∞ *terhadap* T'_∞ *min suhu, iaitu suhu bahantara di sekeliling silinder. Silinder tersebut direndam di dalam bendalir yang mematuhi hukum Newton atau di dalam medium berongga. Penyelesaian beranalisis yang merujuk kepada kaedah kembangan asimptot telah digunakan. Kaedah pemisahan pada kembangan suhu dan fungsi arus bertertib dua diantara yang mantap dengan tak mantap dan penggunaan tatatanda kompleks, memperlihatkan kaedah penyelesaian yang lebih ringkas dan mudah. Penyelesaian diperoleh pada lapisan sempadan dalaman (berdekatan dengan silinder) dan lapisan sempadan luaran (jauh dari silinder). Sebagaimana yang diperoleh oleh pengkaji sebelumnya, suhu dan fungsi arus yang bersifat mantap didapati wujud di luar lapisan sempadan dalaman. Dengan itu, syarat sempadan tidak dipenuhi. Walau bagaimanapun, keputusan yang berlainan diperoleh pada halaju mantap bagi nombor Prandtl,* $P = 1$, *iaitu ianya memenuhi syarat sempadan di luar lapisan sempadan dalaman. Lapisan sempadan luaran telah diselesaikan menggunakan kaedah Fettis terubahsuai. Namun, dengan kesahihan kaedah ini, penyelesaian hanya benar pada titik berdekatan dengan titik genangan. Seterusnya, didapati nombor Reynolds yang besar adalah penting bagi kewujudan aliran mantap.*

Abstract

This thesis is concerned with the problems of natural convection about an infinite horizontal circular cylinder. The temperature at the surface of the cylinder performs harmonic oscillations with frequency w and amplitude bT'_∞ about the mean temperature T'_∞, the temperature of the ambient medium. The cylinder is immersed in a Newtonian fluid or in a porous medium. The analytical technique was performed in order to solve the problems by the method of matched asymptotic expansions. The separation into unsteady and steady second-order temperature and stream function in the expansions and the use of complex notation are shown to be a simple method in the solution. Solutions have been obtained for the inner boundary-layer (adjacent to the cylinder) and for the outer boundary-layer (far from the cylinder). As shown by the previous studies of earlier authors, a steady velocity and steady temperature persisted outside the thin inner boundary-layer. The boundary conditions at the outer edge of the inner layer could not be satisfied. This phenomenon is analogous to the problem of the flow caused by an oscillating cylinder. However, different result was obtained for the steady velocity at the outer edge of the inner boundary-layer for Prandtl number $P = 1$, in which the steady tangential velocity tends to zero at a distance far from the cylinder. The outer boundary-layer equations have been solved by the modified Fettis method. However, due to the validity of this method, the solutions in the outer region break down at the points which are far from the stagnation point. Further, we found the importance of the large Reynolds number for the existence of the steady flow properties.

Nomenclature

a	typical length of the body or the the radius of the circular cylinder,
b	non-dimensional amplitude in the temperature oscillation,
c_s	specific heat of incompressible solid phase,
c_{Pf}	specific heat at constant pressure of the fluid phase,
C_1 and C_2	empirical constants that depend on the chosen length scale,
c_F	a dimensionless constant,
D_p	pore size,
$\underline{F}$	vector of the external force per unit mass,
g	magnitude of the acceleration due to gravity,
Gr	Grashof number, $= \beta g(T_c' - T_\infty')a^3/\nu^2$,
$h.o.t.$	higher order terms,
K	thermal conductivity of the fluid,
K_p	thermal conductivity of the porous medium,
K_s	thermal conductivity of the solid phase,
l_0	geometric reference length of the body,
Nu	Nusselt number, $= C_1(Ra)^{C_2}$,
$O(x)$	Order x,
P	Prandtl number, $= \nu/K$,
p'	pressure of the fluid,
p	non-dimensional pressure of the fluid,
p_∞'	constant pressure of the fluid at large distance from the cylinder,
q	real number,

$\underline{q}'$	dimensional velocity in (r', θ) directions,
$\underline{q}$	nondimensional velocity in (r, θ) directions,
r'	distance measured from the axis of the cylinder,
r	non-dimensional distance measured from the axis of the cylinder,
Ra	Rayleigh number, $= \frac{U_c a}{\alpha}$
Re_p	Reynolds number based on typical pore diameter, $= \frac{V_p d_p}{\nu}$,
R	Reynolds number, $= \frac{U_c a}{\nu}$,
S	Strouhal number, $= 1/\epsilon$,
t'	time,
t	non-dimensional time, $= \omega t'$,
t_0'	reference time,
T_m'	temperature above T_∞',
T_∞'	temperature of the ambient fluid or at infinity or mean temperature of surrounding fluid or of porous medium,
T'	temperature of the fluid,
T_c'	temperature of the surface of the body,
T	non-dimensional temperature of the cylinder, $= (T_c' - T_\infty')/bT_\infty'$
$T_x^{(i)}$	temperature of order x in the inner region,
$T_x^{(i)s}$	steady temperature of order x in the inner region,
$T_x^{(o)}$	temperature of order x in the outer region,
(u', v')	fluid velocity components in (r', θ) directions,
(u, v)	non-dimensional fluid velocity components in (r, θ) directions,
U_c	velocity scale based on the maximum speed of the oscillation of the cylinder or the surface temperature of the cylinder in Newtonian fluid or in a porous medium.

V_p corresponding pore velocity scale, $= \frac{V_v}{\Phi}$

V_v volume average vertical velocity scale, .

z' distance measured normal to the surface of the body,

Greek

α effective thermal diffusivity,

β coefficient of the thermal expansion of the fluid,

δ measure of diffusion distance,

δ_d distance from the surface of the body or the thickness of a thin layer of vorticity or boundary-layer,

Δ difference between two properties,

ϵ inverse of Strouhal number, $= 1/(\gamma R) = S/R = \frac{U_c}{a\omega}$ in a Newtonian fluid or $1/(\gamma Ra) = \frac{U_c}{a\omega}$ in a porous medium,

η scaled inner boundary-layer coordinate in a Newtonian fluid and in a porous medium, $= \frac{r-1}{\epsilon\sqrt{2\gamma}}$

$\bar{\eta}$ scaled outer boundary-layer coordinate, $= \sqrt{\frac{P}{\epsilon\gamma}}(r-1)$ in a Newtonian fluid or $\frac{1}{\sqrt{\epsilon}}(r-1)$ in a porous medium,

γ non-dimensional parameter, $= 1/(\epsilon R) = S/R = \nu\omega/U_c^2$ or $1/(\epsilon Ra)$ $= S/Ra = K\omega/U_c^2$, in a Newtonian fluid or in a porous medium, respectively,

κ permeability of porous medium,

λ either real or complex number,

μ dynamic viscosity of the fluid,

$\widetilde{\mu}$ an effective viscosity,

ν coefficient of kinematic viscosity of the fluid,$=\mu/\rho$,

∇^2	Laplace operator,
ϕ_w	angle measured clockwise from the horizontal,
Φ	porosity of the porous medium,
ψ	stream function,
$\psi_x^{(i)}$	stream function of order x in the inner region,
$\psi_x^{(i)s}$	steady stream function of order x in the inner region,
$\psi_x^{(o)}$	stream function of order x in the outer region,
ρ_s	density of the solid,
ρ	density of the fluid,
ρ_∞	density of the ambient fluid,
σ	ratio of heat capacity of the saturated porous medium to that of the fluid,
θ	angle measured anticlockwise from the downward vertical,
$\xi(\theta)$	function of θ only,
ω	frequency of the oscillation,

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Chapter 1

INTRODUCTION

1.1 Problem Nature

Our problem is related to the natural convection about an infinitely long horizontal circular cylinder, thus a two-dimensional cylindrical problem, which is immersed in a stationary fluid or porous medium. The surface of the cylinder is oscillating harmonically for an infinitely long time about the temperature of ambient fluid or porous medium. The domain of the solution is for the whole region of the cylinder that is near a stagnation point, which is divided into two boundary-layers, an inner boundary-layer and an outer boundary-layer. The solution in the inner boundary-layer is obtained for the unsteady and steady temperature and stream function, while only the steady temperature and stream function are considered in the outer region. This is an extension of the work of Roslan (2002a), whose notations and explanations are closely followed. In this thesis we use a new method of solution. Our discussions are started with a general introduction to heat transfer, then specifically to the problem in a Newtonian fluid and in a porous media.

1.2 Objectives

An objectives in this study are based on some arguments described here. It is postulated that, the method of separation of the steady and the unsteady components of flow field presented by (Chatterjee & Debnath, 1979) from the governing equations can be used without encountering many difficulties. Another approach is without any separation been done on the governing equations as in the work by (Merkin, 1967; Roslan *et al.* 2003a, 2003b, 2004a). However, instead of separating the stream function ψ and the temperature T from the governing equations, another expansion approach is by separating the particular term, i.e. the second term in the expansions of ψ and T, as the steady and unsteady parts as was done by (Riley, 1967) in the problem of oscillating cylinder.

Schlichting, (1932) employed a complex notation in the solution of convection problem over an oscillating cylinder. Since the differential equations are linear, the calculations performed by (Schlichting, 1932) did not have difficulties on the appearance of an extra real part when taking the real part of the product of two complex quantities. However, the governing equations that will be considered in our problems contain nonlinear terms. Obviously, the product of two complex quantities, which are either a product of the stream function by the temperature or by the derivative of stream function will produce another real term coming from the product of two imaginary terms.

The previous studies such as, (Schlichting, 1932; Riley, 1967) found that a steady streaming flow was discovered in the problem of periodic boundary-layer flow on a circular cylinder oscillating harmonically in a direction perpendicular to the axis of the cylinder in an otherwise stationary ambient medium. The steady flow was also found in the problem of oscillating natural convection about a horizontal circular cylinder see, (Merkin, 1967; Chatterjee & Debnath, 1979). Due to the existence of the steady flow either the steady temperature or the steady stream function in the outer boundary-layer, we will perform an analysis regarding the comparative magnitude of the significant quantities that will influence the flow at the outer edge of the inner boundary-layer.

Thus, the objectives of this work based on the previous studies are as follows:

(i) Present an alternative simple approach for solving the oscillating natural convection in the inner boundary-layer.

(ii) Propose an appropriate way to avoid an error from a direct implementation of Euler's formula. The investigation of this effect in the solution of the oscillatory natural convection problems will be presented in both medium.

(iii) Investigate the importance and the existence of the steady flow for the problem of natural convection about a horizontal circular cylinder when the temperature of the cylinder surface is oscillating harmonically analogous to the oscillating cylinder.

(iv) Produce the appropriate value of Reynolds number that is needed to be considered for the problem of the oscillatory natural convection about a circular cylinder.

1.3 Introduction to Heat Transfer

A study of considerable importance in areas concerned with the physical processes involved in energy generation and its utilization is a concern of heat transfer. With the growing sophistication in technology and with the increasing concern with energy and the environment, the study of heat transfer has, over the past few years, been related to a very wide variety of problems, each with its own demands of precision and elaboration in the understanding of particular processes of interest. Areas of study range from atmospherical, geophysical, technical and environmental problems, to those in heat rejection, space research and manufacturing systems.

Within thermal energy transport are the two basic processes of conduction and radiation, with the former due to the motion of the microscopic particles that comprise the material when a temperature difference exists in a material. The local motion of a particle is dependent on the local temperature in the material and diffusion of energy occurs due to differences in this local motion. The energy transfer in the later mode, radiation, is in the form of electromagnetic waves. Energy is emitted from a material due to its temperature level and is then transmitted to another surface through the intervening space, which may be a vacuum or a medium which may absorb, reflect or transmit the radiation depending on the nature and extent of the medium.

A third mode of heat transfer which will be a topic in this discussion is convection, in which the conductive heat transfer process is coupled with the motion of the fluid. As a consequence of this fluid motion, the heat transfer rate, as given by conduction, is often considerably modified. The relative motion of the fluid provides an additional mechanism for the transfer of energy and of the material. the latter being an important consideration in cases where mass transfer, due to a concentration differences, occurs. Convection is inevitably coupled with the conductive mechanisms since, although the fluid motion modifies the transport process, the eventual transfer of energy from one fluid element to another in its neighbourhood is through conduction. Also, at a fluid or solid interface, the process is predominantly conduction due to the relative fluid motion being brought to zero there. Therefore, a study of convective heat transfer involves the mechanisms of conduction and, sometimes, those of radiative processes as well, coupled with those of fluid flow. This makes the study of this mode of heat, or mass, transfer a very complex one, and its importance in technology and in nature can hardly be exaggerated.

Convective heat transfer is further divided into two basic processes. If the motion of the fluid arises due to an external agent, such as an externally-imposed flow of a fluid stream over a heated object, the process is termed as forced convection. The motion may be the result of, for example, a fan, a blower, the wind, a suction device, or the motion of the heated object itself. Such problems are frequently encountered in technology where the heat transfer to, or from, a body is often due to an imposed flow of a fluid at a temperature which is different from that of the body. If, on the other hand, no such externally-induced flow is provided and the flow arises naturally simply due to the effect of density differences in the gravitational force field, resulting from temperature or concentration differences, the process is referred to as natural convection. The density differences give rise to buoyancy effects due to which the flow is generated. A heated body cooling in ambient fluid generates such a flow in the region surrounding it. Similarly, the buoyant flow arising from heat rejection to the atmosphere and to other ambient media, and circulations arising around the heated bodies, can give rise to thermal stratification of the medium, as in temperature inversion, and many other such heat transfer processes, in our natural environment. The flow may also arise due to concentration differences, such as those caused by the saline differences in the sea and by composition differences in chemical processing units and thus cause a natural convection of heat and mass transfer.

The main difference between natural and forced convection lies in the very nature of the flow generation. In forced convection, the externally imposed flow is, in general, known, whereas in natural convection, the flow results from an interaction of the density with the gravitational, or some other body, force field and is therefore invariably linked with, and dependent on, the temperature and concentration fields. As such, the motion that arises is not known at the onset and has to be determined from a consideration of the heat and mass transfer processes coupled with fluid flow mechanisms. In general, natural convection velocity levels are much smaller than those encountered in forced convection. In fact, any heated body which is subjected to an external flow will give rise to natural convection effects due to the difference between its temperature and that of the neighbouring fluid. In many cases of practical interest both processes are important and the heat transfer is by mixed convection in which neither mode is truly predominant.

The main difference between the two mechanisms really lies in the word external. A heated body lying in an otherwise still fluid loses energy by natural convection. However, as it does so it also generates a buoyant flow above it and so when another body is placed in that flow it is subjected to an external flow. Hence it becomes necessary to determine the natural convection effects, as well as the forced convection effects, and the regime in which the heat transfer mechanisms lie. However, in our later applications we will consider the situation of natural convection as opposed to forced or mixed convection. The above differences between natural and forced convection make the analysis, as well as the experimentation, of processes involving natural convection generally much more complicated than those in forced convection. Special techniques and methods have therefore to be devised, with a view to provide information on the flow and on the heat and mass transfer rates. We will present these problems later in this chapter in our review of previous work.

The study of natural convection heat transfer in interacting flow fields has previously received much attention, not only theoretically, by way of analytical or numerical methods, but also experimentally, due to the importance of the influence of fluid flow and the heat transfer configurations. The existence of a temperature gradient in the surrounding fluid may cause a flow induced by the buoyancy force. Such a flow, usually called natural convection or free convection, will distort the previous temperature distribution and modify the overall heat transfer rate across the boundary. In this thesis we consider the problem of natural convection for the circular cylinder.

The basic conclusions of all the researchers working in convective flows can be summarized as follows. Convective flows have a variety of features depending upon:

(i) the properties of the medium,

(ii) the surrounding conditions,

(iii) the geometrical configurations.

The analysis of the fluid flow and the heat transfer is usually based on transport equations derived from differential continuum laws, i.e. continuity equation, momentum equation, and energy equation; together with the boundary conditions which can be analyzed:

(i) theoretically: analytical or numerical or both

(ii) experimentally,

(iii) theoretically and experimentally.

The solution we seek is based on the stream function or the velocity field, the temperature distribution or the heat transfer rate, and some governing parameters. Next, we will discuss natural convection in a Newtonian fluid and in a porous medium separately.

1.4 Introduction to Natural Convection in a Newtonian Fluid

Natural convection is a consequence of heat transfer that arises over the surface of a body when it is at a temperature different from that of the ambient medium. In the case, of a heated body cooling in an extensive isothermal medium, the fluid flows adjacent to the hot surface of the body and the heated fluid eventually rises above the body as a buoyant flow or wake. Similarly, a body colder than the ambient fluid would cause a flow opposite to that due to a heated body, since the fluid adjacent to the body becomes colder and, hence, heavier than the ambient fluid, resulting in a flow in the direction of the gravitional force. In nature too, many natural convection flows occur adjacent to heated or cooled surfaces, such as those arising over the human body and over the surface of a lake due to the temperature differences that exist. Instead of a body being suddenly heated or suddenly cooled, the temperature can oscillate harmonically or inharmonically with time.

The body itself may be flat, such as a plate or wall, or curved, such as sphere, wire or circular cylinder. It may also be vertical, horizontal or inclined. In this study, we will investigate the natural convection over an horizontal circular cylinder when the temperature is periodic and whether or not steady flows are reached for large time.

1.4.1 Introduction to Boussinesq Approximation

In solving the problem of natural convection boundary-layer flows, an important approximation to be used is the Boussinesq approximation, which is an essential of the gravity acceleration in producing the buoyancy term as the body force of the flow when the temperature variations are not too large (Illingworth, 1949; Spiegel & Veronis, 1960; Chandrasekhar, 1961; Chow, 1979). The Boussinesq approximation has now become a very important principle and has been used in various studies by numerous investigators.

Although this approximation is generally attributed to (Boussinesq, 1903), it is known from the work of (Mihaljan, 1962) that the idea was first presented by (Oberbeck, 1879, 1891) in his meteorological studies and thus the approximation is also referred to as the Oberbeck-Boussinesq approximation. In some respects, (Ober-