USING THE LINE ARRAY REPRESENTATION AS AN ALTERNATIVE METHOD FOR UNDERSTANDING WHOLE NUMBER MULTIPLICATION AMONG PRIMARY STUDENTS

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FACULTY OF PSYCHOLOGY AND EDUCATION
UNIVERSITI MALAYSIA SABAH
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THIS THESIS IS SUBMITTED IN FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

FACULTY OF PSYCHOLOGY AND EDUCATION
UNIVERSITI MALAYSIA SABAH
2015

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DECLARATION

I hereby declare that the material in this thesis is my own except for quotations, excerpts, equations, summaries and references, which have been duly acknowledged.

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DEGREE : **DOCTOR OF PHILOSOPHY (MATHEMATICS EDUCATION)**

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ACKNOWLEDGEMENTS

First and foremost, I will thank Lord, the Almighty Father, being so compassionate and merciful has kindly helped me to complete my thesis.I truly believe my courage and energy for undertaking as well as completing my Ph.D successfully was only possible due to the Glory of God.

The success of this thesis is attributed to the extensive support and assistance from my supervisors, Associate Professor Dr. Tan Choon Keong, Professor Dr. Vincent Pang and Dr. Sporty Chin. With that, I would like to express my most grateful gratitude and sincere appreciation to all of them for their guidance, valuable advice, supervision, encouragement and kindness to me throughout this study.

First of all, I would like to thank Associate Professor Dr. Tan Choon Keong, my main supervisor for he has set high expectations for me to meet and challenged me to be an independent thinker. His constructive criticisms were always timely and helpful in the write up of my research methodology.

I would like to thank Professor Dr. Vincent Pang for patiently led me through a rather complex quantitative analysis using the Rasch Model. I really appreciate very much his effort of organising and inviting me twice to a short course on Rasch Model – absolutely free of charge. Thank you also for the prompt feedbacks, editing and proof reading of my thesis and articles written for publications.

I would like thank Dr. Sporty Chin, whom I met in 2012 for spending quality time, sharing ideas on basic mathematics operations, introducing me to David Tall's (his Ph.D supervisor at the University of Warwick, U.K.) webpage and guiding me through the qualitative analysis. Thank you so much for graciously helping me to obtain articles from journals during his study in the U.K. which I couldn't access from here.

My sincere thanks to the external examiner, Associate Professor Dr. Chew Cheng Meng as well as to both of my internal examiners, Associate Professor Dr. Mohd. Zaki bin Ishak and Dr. Siew Nyet Moi @ Sopiah Abdullah for their remarkable views, comments and suggestions that have further improved the overall flow of ideas throughout the five chapters.

I would like to express my special thanks to Dr. Patrick Barmby, who is presently working as a Senior Lecturer in Mathematics Education at University of the Witwatersrand (formerly at University of Durham, U.K.) for sharing ideas on primary mathematics, focusing in array representation. My most sincere thanks also goes to Associate Professor Dr. Ho Chong Mun, an expert in recreational mathematics working at Universiti Malaysia Sabah for his kind contributions to further improve my research tools. Thank you very much to both of you for validating my tools and the test.

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A special thank you I wish to the Heads; especially Datin Hajah Florence Faizah Kouju, teachers and students of all the primary schools for the fullest cooperation given to me throughout the interventions during the pilot and main study. I am truly also indebted to a group of my undergraduates students for helping me in the collection of test data from eight primary schools in the district of Tawau during the pilot study. Thank you very much to my ex-student, Miss Lim Ai Teng for preparing the devices for audio- and video-recordings and helping in the collection and transcription of the audiovisual data during the main study. I am forever grateful to all the wonderful and cheerful students (in the pilot and main study) for their active participations and sincere cooperation given to me.

Last but least, to my family, especially my husband, daughter, son, mother, father, sisters, brother, nieces and nephews as well as to all my friends, especially Noraini Said, Idarmawati Ahmad and Israh Ariffin and every other people who are not listed above, thank you so much for their unwavering support, continuous encouragement, consistent prayers and great patience throughout my long journey towards the successful completion of my study. They have made my Ph.D process a pleasure of learning and gaining more knowledge.

Chai Chin Pheng

20 August 2015

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ABSTRACT

This study aims to examine how Line Array Representation can support primary school students' understanding in single digit multiplication (SDM) and multi-digit multiplication (MDM) whole number calculations among Year 3 and Year 5 students as well as to determine its effect on the students' understanding of whole number multiplication through a multiplication test for each level constructed based on the range of calculation strategies associated with multiplication. A theoretical framework in mathematical growth which connects between the embodiment and symbolism domains was developed to examine the understanding in multiplication.The construction of multiplication concept can be examined through whether student is able to perceive, act and reason upon the line array that is considered to be the key representation for multiplication in this study. Having established the theoretical framework, the research design consists of a mixed methods approach was drawn up to conduct a qualitative study in the first phase and followed by a quantitative study in the second phase. The design is based on 'QUAL → quan Sequential Exploratory Design' which has given priority to the qualitative aspect of the study. The participants of this study were taken from an intact class of 29 Year 3 (15 boys, 14 girls) and 32 Year 5 (19 boys, 13 girls). Year 5 data was collected during an intervention in early February to mid-March 2014 while Year 3 data in mid-March to end of April 2014. Qualitative data was collected by using the teaching experiment technique. Students worked in pairs on ten SDM and two MDM tasks. Analysis of the obtained audio- and video-recordings, written tasks and field notes had identified strategies in which the Line Array Representation supported or hindered students' multiplication calculations. Results showed that students were able to apply the commutative and distributive over addition property to solve multiplication problem, most evident in 2-digit by 2-digit multiplication. Using line array, students were capable of inventing own procedures, solving MDM from the left to the right (thousands, hundreds, tens and units) obviously opposes the traditional algorithm of working from right to left. The quantitative data was collected from the 33-item multiplication test which was administered to each student before and after the intervention. The data obtained from the tests were subjected to Rasch analysis using Quest software. Overall, the resulting measures have shown to have good reliability as indicated by a Cronbach's alpha value of 0.79 for Year 3 and 0.82 for Year 5. The higher mean person estimates (Year 3 = +0.27 logits and Year 5 = +0.98 logits) compared with the mean item threshold defaulted at 0.00 logit showed that items were likely well matched for Year 3 students and easy for Year 5 students. Problems that students had faced with using the array and dilemmas that researcher encountered throughout the study are discussed. Based on these findings, two implications of this study are (1) pedagogical use of visual representations by teachers and (2) the benefits of using Item-Response Theory (IRT) model by teachers in assessments. Recommendations for further improvements in the test, structure of the line array and a different method of data collection are also put forward.

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ABSTRAK

MENGGUNAKAN PERWAKILAN SUSUNAN GARISAN SEBAGAI KAEDAH ALTERNATIF DALAM KEFAHAMAN PENDARABAN NOMBOR BULAT DALAM KALANGAN MURID SEKOLAH RENDAH

Kajian ini bertujuan untuk meneliti bagaimana perwakilan susunan garisan boleh menyokong murid dalam pendaraban nombor bulat satu digit dengan satu digit (SDM) dan pendaraban berbilang digit (MDM) murid Tahun 3 dan Tahun 5 serta untuk menentukan kesannya terhadap kefahaman murid tersebut tentang pendaraban nombor bulat melalui ujian Pendaraban yang dibina bagi setiap tahap berdasarkan pelbagai strategi pengiraan yang berkaitan dengan pendaraban. Dengan mengadaptasikan kerangka teori yang melibatkan perkembangan matematik, kefahaman konsep pendaraban bagi murid sekolah rendah dapat dilihat sebagai persilangan di antara domain penjelmaan dan simbolisme dan di mana strategi pengiraan pendaraban akan muncul di dalam persilangan kedua-dua domain tersebut. Pembinaan konsep pendaraban boleh dikaji melalui sama ada murid dapat melihat, bertindak dan menaakul; perwakilan susunan garisan yang dianggap sebagai perwakilan utama bagi pendaraban dalam kajian ini. Setelah mengukuhkan kerangka teori, rekabentuk penyelidikan yang terdiri daripada pendekatan kaedah gabungan digunakan untuk menjalankan satu kajian kualitatif dalam fasa pertama dan diikuti oleh kajian kuantitatif dalam fasa kedua. Rekabentuk kajian adalah berdasarkan rekabentuk eksploratori bersiri 'QUAL►quan' di mana keutamaan analisis data diberi kepada aspek kualitatif dalam kajian ini. Peserta kajian ini terdiri daripada satu kelas Tahun 3 seramai 29 orang (15 lelaki, 14 perempuan) dan satu kelas Tahun 5 seramai 32 orang murid (19 lelaki, 13 perempuan), di mana tahap kebolehan murid telah dibahagikan kepada tiga tahap yang berbeza; tinggi, sederhana dan rendah. Data Tahun 5 dikumpul dari awal Februari hingga pertengahan Mac 2014 manakala data Tahun 3 dari pertengahan Mac hingga akhir April 2014 melalui intervensi seperti yang dirancangkan. Data kualitatif dikumpul dengan menggunakan teknik Eksperimen Pengajaran, di mana murid secara berpasangan menyelesaikan tugasan yang terdiri sepuluh sepuluh SDM dan dua MDM tugasan dalam lembaran kerja yang disediakan. Analisis daripada data kualitatif yang diperolehi secara audiovisual dan bertulis telah mengenalpasti beberapa strategi yang digunakan oleh murid menunjukkan perwakilan susunan garisan dapat menyokong murid dalam pengiraan pendaraban. Dapatan kajian menunjukkan bahawa murid mampu mengaplikasikan Hukum Tukar Tertib dan Taburan dalam penambahan untuk menyelesaikan masalah pendaraban, paling ketara ditunjukkan dalam pendaraban 2-digit dengan 2-digit. Menggunakan perwakilan garisan tersebut, murid berupaya mencipta prosedur terutama dalam menyelesaikan MDM, iaitu mengira dari kiri ke kanan (ribu, ratus, puluh dan sa). Cara pengiraan ini jelas menunjukkan ia bertentang dengan algoritma yang tradisional, iaitu bekerja dari kanan ke kiri. Data kuantitatif pula dikutip daripada skor ujian pendaraban yang terdiri daripada 33 item telah ditadbirkan kepada setiap murid sebelum dan selepas intervensi. Data yang diperolehi daripada ujian-ujian tersebut telah dianalisis dengan menggunakan analisis Rasch, perisian Quest. Secara keseluruhan, dapatan kajian menunjukkan

kebolehpercayaan item yang baik, di mana nilai alfa Cronbach adalah 0.79 untuk Tahun 3 dan 0.82 untuk Tahun 5. Min anggaran kebolehan murid yang lebih tinggi (Tahun 3 = +0.27logits dan Tahun 5 = +0.98 logits) berbanding dengan min item (0.00 logit) menunjukkan kemungkinan bahawa item-item adalah berpadanan dengan tahap kebolehan murid Tahun 3 manakala agak mudah untuk murid Tahun 5. Tambahan pula, semua peta 'Item-fit' untuk Tahun 3 (kecuali 3 item) and Tahun 5 menunjukkan semua item adalah 'fit' dalam julat 0.7 ke 1.3. Kajian ini membincangkan masalah yang telah dihadapi oleh murid semasa menggunakan perwakilan susunan garisan dan dilema yang dihadapi oleh penyelidik semasa melaksanakan kajian. Berdasarkan dapatan kajian ini, terdapat dua implikasi utama iaitu (1) menggunakan perwakilan secara visual dalam pedagogi oleh guru-guru dan (2) faedah-faedah menggunakan model Item-Response Theory (IRT) oleh guru-guru dalam penilaian. Cadangan-cadangan untuk penambahbaikan item dalam ujian, struktur perwakilan susunan garisan dan cara mengutip data yang berlainan juga telah dikemukakan.

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UMS
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LIST OF SYMBOLS / ABBREVIATIONS

es	effect size
EPPRD	Education Policy Planning and Research Division
H	Hundreds
HOTS	Higher Order Thinking Skills
Infit MNSQ	Infit Mean Square
IRT	Item-Response Theory
IPM	Item-Person Map
IPMs	Item-Person Maps
IU	Instrumental Understanding
JPNS	Sabah State Education Department
KBSR	Kurikulum Bersepadu Sekolah Rendah
KPM	Kementerian Pendidikan Malaysia
KSSR	Kurikulum Standard Sekolah Rendah
LOTS	Lower Order Thinking Skills
MDM	Multi-Digit Multiplication
MOE	Ministry Of Education
MMR	Mixed Methods Research
Outfit MNSQ	Outfit Mean Square
RU	Relational Understanding
SDM	Single Digit Multiplication
T	Tens
Th	Thousands
U	Units / ones
2D x 1D numbers	multiplication of 2-digit with 1-digit whole numbers
2D x 2D numbers	multiplication of 2-digit with 2-digit whole numbers

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