WEIGHTED MEAN ITERATIVE METHODS FOR SOLVING FREDHOLM INTEGRAL EQUATIONS

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2012

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THESIS SUBMITTED IN FULFILLMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

SCHOOL OF SCIENCE AND TECHNOLOGY
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2012

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DEGREE : **DOCTOR OF PHILOSOPHY (NUMERICAL ANALYSIS)**

VIVA DATE : **26th March 2012**

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ACKNOWLEDGEMENT

It is my pleasure to express my sincere thanks to all who have helped me during my time in Universiti Malaysia Sabah and made this thesis possible.

First of all, I wish to express my deepest gratitude and appreciation to my supervisor, Associate Professor Dr. Jumat Sulaiman for his valuable full hearted guidance, confidence and constant encouragement throughout my time as a PhD student.

I would also like to express my love and gratitude to my parents, Muthuvalu and Anchalai Deve, for their understanding and patience throughout the duration of my studies.

Special thanks to all my fellow friends for their help and moral support during my time in Universiti Malaysia Sabah.

Finally, I acknowledge the Postgraduate Research Grant, Universiti Malaysia Sabah (GPS0003-SG-1/2009) and Skim Bantuan Pascasiswazah, Universiti Malaysia Sabah for the financial supports.

ABSTRACT

WEIGHTED MEAN ITERATIVE METHODS FOR SOLVING FREDHOLM INTEGRAL EQUATIONS

Integral equations (IEs) are used as mathematical models for many and varied physical circumstances, and also occur as reformulations of other mathematical problems. In this research, first and second kind linear IEs of Fredholm type are considered and solved using numerical approaches. The essential aim of this research was to investigate the effectiveness of the point and block Weighted Mean (WM) iterative methods categorized as two-stage iterative methods in solving linear systems generated from the discretization of the first and second kind linear Fredholm integral equations (FIEs). In the aspect of discretization schemes, three schemes of different order under composite closed Newton-Cotes quadrature and piecewise polynomial collocation methods were used to discretize first and second kind linear FIEs. Moreover, discussions on computational complexity of the tested point and block WM methods in this research were also included. By comparing point WM iterative methods, the point methods under Geometric Mean (GM) and Harmonic Mean (HM) families are slightly superior to equivalent Arithmetic Mean (AM) methods, particularly for first kind linear FIEs. Meanwhile, performance of the point GM and HM methods is comparable. Based on numerical experiments, results show that proposed 6-Point Quarter-Sweep Block Arithmetic Mean (6-QSBLAM), 6-Point Quarter-Sweep Block Geometric Mean (6-QSBLGM) and 6-Point Quarter-Sweep Block Harmonic Mean (6-QSBLHM) methods are the best tested AM, GM and HM iterative methods respectively in solving composite closed Newton-Cotes quadrature and piecewise polynomial collocation systems associated with numerical solutions of first and second kind linear FIEs in the sense of number of iterations and CPU time. For comparison purpose among 6-Point Quarter-Sweep Block Weighted Mean (6-QSBLWM) methods, 6-QSBLGM and 6-QSBLHM methods are slightly better than 6-QSBLAM method in solving FIEs. All variants of point and block WM methods, which were formulated using the half- and quarter-sweep iteration concepts reduce the computational complexity of the standard WM iterative methods at least 75% and 93.75% respectively. In terms of accuracy, all three schemes under piecewise polynomial collocation method yields more accurate approximation solutions than composite closed Newton-Cotes quadrature schemes particularly for the first kind FIEs problems. However, by comparing corresponding orders of composite closed Newton-Cotes quadrature and piecewise polynomial collocation schemes, the accuracy of the approximation solutions is comparable when solving second kind linear FIEs.

ABSTRAK

Persamaan kamiran (IEs) digunakan sebagai model matematik untuk memperihalkan pelbagai keadaan fizikal, dan juga wujud dalam perumusan semula masalah-masalah matematik yang lain. Dalam kajian ini, persamaan kamiran Fredholm (FIEs) linear jenis pertama dan kedua dipertimbangkan dan diselesaikan dengan menggunakan pendekatan berangka. Tujuan utama kajian ini adalah untuk mengkaji keberkesanan kaedah lelaran titik dan blok Min Berpemberat (WM) yang juga dikategorikan sebagai kaedah lelaran dua tahap dalam menyelesaikan sistem persamaan linear yang dijana daripada pendiskretan FIEs linear jenis pertama dan kedua. Di dalam aspek skema pendiskretan, tiga skema dengan peringkat yang berbeza bagi kuadratur Newton-Cotes tertutup gubahan dan penempatan bersama polinomial cebis demi cebis digunakan untuk mendiskret masalah FIEs linear jenis pertama dan kedua. Selanjutnya, perbincangan mengenai kekompleksan pengiraan bagi kaedah titik dan blok WM yang dikaji di dalam kajian ini juga dimuatkan. Dengan membandingkan kaedah-kaedah lelaran titik WM, kaedah-kaedah lelaran titik dari famili Min Geometri (GM) dan Min Harmonik (HM) adalah lebih baik daripada kaedah Min Aritmetik (AM) yang sepadan, terutamannya bagi masalah FIEs linear jenis pertama. Sementara itu, pelaksanaan kaedah-kaedah GM and HM adalah setanding. Berdasarkan ujikaji berangka, keputusan menunjukkan bahawa kaedah usulan 6-Titik Blok Min Aritmetik Sapuan Sukuan (6-QSBLAM), 6-Titik Blok Min Geometri Sapuan Sukuan (6-QSBLGM) and 6-Titik Blok Min Harmonik Sapuan Sukuan (6-QSBLHM) merupakan kaedah AM, GM dan HM masing-masing yang paling efektif bagi menyelesaikan sistem kuadratur Newton-Cotes tertutup gubahan dan penempatan bersama polinomial cebis demi cebis yang berhubungkait dengan penyelesaian berangka FIEs linear jenis pertama dan kedua apabila kriteria bilangan lelaran dan masa CPU dipertimbangkan. Dalam membandingkan keberkesanan kaedah-kaedah 6-Titik Blok Min Berpemberat Sapuan Sukuan (6-QSBLWM), didapati bahawa kaedah 6-QSBLGM and 6-QSBLHM adalah lebih baik daripada kaedah 6-QSBLAM bagi menyelesaikan masalah FIEs. Kepelbagaian kaedah titik dan blok WM yang diterbitkan dengan menggunakan konsep lelaran sapuan separuh dan sukuan masing-masing dapat mengurangkan kekompleksan pengiraan bagi kaedah lelaran piawai WM sekurang-kurangnya 75% dan 93.75%. Dalam hal kejituan, kesemua tiga skema penempatan bersama polinomial cebis demi cebis yang diaplikasikan menghasilkan penyelesaian berangka yang lebih jitu jika dibandingkan dengan skema kuardratur Newton-Cotes tertutup gubahan terutamanya bagi masalah FIEs linear jenis pertama. Walaupun demikian, kejituan penyelesaian berangka adalah hampir sama apabila skema-skema kuardratur Newton-Cotes tertutup gubahan dan penempatan bersama polinomial cebis demi cebis yang dibandingkan pada peringkat sepadan bagi masalah FIEs linear jenis kedua.

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