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MR. RAJASEGERAN

Nama Penyelia

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FIBONACCI SEQUENCE AND SHELLS

ANEED ANAND A/L NARANAN @ NARAYANAN

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2007

DECLARATION

I affirm that this dissertation is of my own effort, except for the materials referred to as cited in the reference section.

April 2007

ANEED ANAND NARANAN@NARAYANAN
HS2004-1719

CERTIFIED BY

Signature

1. **SUPERVISOR**

   (Mr. Rajasegeran a/l Ramasamy)

2. **EXAMINER 1**

   (Mr. Victor Tiong Kung Ming)

3. **DEAN**
   (Supt./Ks. Prof. Madya Dr. Shariff A. K. Omang)

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ABSTRACT

Shells are the exoskeleton of certain creatures which include mollusks. Previously a research was conducted to show the relevance of the Fibonacci sequence on the spiral shape on the nautilus shell. This concluded in the usage of the Golden Rectangle which is now widely used to describe many different types of spirals including spirals of galaxies. The Fibonacci sequence is connected to the Golden Ratio therefore this dissertation is not only directly concentrated on the Fibonacci sequence. In this dissertation, a lot of concentration was given to two main types of shells; the gastropod family shells and the bivalve family of shells. These two types of shells are abundantly found on the beaches of Malaysia. Beach combing method was used to collect these shells, which is basically the method of walking by the beach and collecting shells that meets the eye. As for the analysis part, the shells were observed and compared to many shapes to find the relevance of the Fibonacci sequence and the Golden Ratio in these shells. A sample size of 20 shells for each type of shell was used to ascertain that the conclusion would include shells of different size too. As a result, three main diagrams were created to prove the Fibonacci sequence and the Golden Ratio on six different types of shells particularly from the gastropod and bivalve families.

SIRI FIBONACCI DAN CENKERANG

ABSTRAK

Cengkerang merupakan rangka luar bagi organisma yang tidak mempunyai rangka seperti mamalia. Cengkerang paling ketara pada haiwan jenis molluska. Sebelum ini, suatu kajian telah dijalankan untuk menunjukan kewujudan siri Fibonacci pada cenkerang nautilus. Sebagai penyelesaiannya, kajian ini telah menggunakan Golden Rectangle untuk menunjukan kewujudan siri Fibonacci pada cengkerang ini. Kini, Golden Rectangle digunakan untuk menunjukan kewujudan siri Fibonacci pada segala jenis putaran termasuk putaran galaksi dan lain-lain lagi. Siri Fibonacci berkait rapat dengan Golden Ratio, oleh itu, disertasi ini memfokuskan siri Fibonacci dan juga Golden Ratio. Dalam disertasi ini, dua jenis cengekrang digunakan iaitu cenkerang gastropod dan cengkerang bivalve. Dua jenis cengkerang ini adalah jenis cengkerang yang paling senang didapati di Malaysia. Bagi aspek analysis kajian ini, cengkerang yang di pungut di perhati dan di bandingkan dengan pelbagai jenis bentuk untuk mencari kewujudan siri Fibonacci dan Golden Ratio pada cengkerang tersebut. Sejumlah 20 cengkerang daripada setiap jenis cengkerang digunakan supaya kesimpulan kajian ini meliputi pelbagai saiz cengkerang. Sebagai kesimpulan, tiga jenis metodologi di temui untuk menunjukkan siri Fibonacci pada enam jenis cengkerang yang didapati daripada famili gastropod dan bivalve.

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LIST OF SYMBOLS

$=$	Equal sign
$\rightarrow$	Arrow
$\geq$	Greater than or equal to
$+$	Addition sign
$-$	Subtraction sign
$/$	Division sign
$\sqrt{\ }$	Square root sign

CHAPTER 1

INTRODUCTION

1.1 HISTORY

1.1.1 The Life Of Fibonacci

If you have read the book by best selling author, Dan Brown, The Da Vinci Code, you probably would already have a brief introduction to the Fibonacci Sequence. Though most of you maybe awed by the fact that mathematical work can be related to even the beauty of a particular person, this thesis will show you more relations of the Fibonacci Sequence and Fibonacci Numbers in our daily lives and hopefully mathematics will be appreciated more. In his book, Dan Brown related the Golden Ratio (one of the products of The Fibonacci Sequence) to the drawing of Leonardo Da Vinci, the Mona Lisa. It is stated that her face follows the Golden Section making her a beautiful person, mathematically speaking. Well, that is not the only use Fibonacci's work. The Fibonacci Sequence, the Fibonacci Numbers and the Golden Section have also been used widely in the fields of music, architecture, art and physical nature.

Fibonacci was born in the 1170's as Leonardo later known as Leonardo Pisano or Leonardo of Pisa since he was born in Pisa, Italy. Later he adopted the more professional name, Fibonacci which meant Filius Bonacci or in English, Son of Bonacci.

Fibonacci's father, Guilielmo Bonacci, worked as a customs officer in Bugia (now called Bougie), North Africa, where he summoned Fibonacci at a young age in preparation of giving his son a good education. Guilielmo must have had great foresight since the effect of Fibonacci's education has paid off to the world for centuries and centuries to come. During Fibonacci's travels with his father, he came to learn and love mathematics.

Fibonacci grew up with a North African education under the Moors (medieval Muslim inhabitants of al-Andalus) and later travelled extensively around the Mediterranean coast. He then met with many merchants and learned their systems of doing arithmetic. He soon realized the many advantages of the "Hindu-Arabic" system over all the others. One of the initial capturing ideas that intrigued Fibonacci was the nine digit number system used in the mathematics of the Indians. He was one of the first people to introduce the Hindu-Arabic number system into Europe, the system we now use today based on ten digits with its decimal point and a symbol for zero: 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. His book on how to do arithmetic in the decimal system, called Liber Abbaci (meaning Book of the Abacus or Book of Calculation) completed in 1202 persuaded many of the European mathematicians of his day to use his "new" system (Knott, 2001).

1.1.2 Fibonacci's Achievements

In the west, Fibonacci was known as the "greatest European mathematician of the middle ages." He died in the 1240's. There is now a statue commemorating him located at the Leaning Tower end of the cemetery next to the Cathedral in Pisa. There are two streets named after Fibonacci, the Quayside Lungarno Fibonacci in Pisa and the Via Fibonacci in Florence.

The most famous problem poised by Fibonacci during his time was the Fibonacci's rabbit problem (*A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?)*. This was in the Liber Abbaci and it was a problem that he faced in 1202.

His achievements, during his lifetime of discoveries, include the writing of the *Liber Abbaci* (the book of calculation), *Liber Quadratorum* (the book of square numbers), this book demonstrated Fibonacci's ease at solving the problems of Diophantus, the greatest mathematician before him, *Practica Geometriae* (the practice of geometry), *Flos* (where he solved many problems at the imperial court) and his most important achievements were his role in bringing Eastern mathematics into Western mathematics and introducing the fractional bar that we use in modern day mathematics.

Fibonacci's works are still famous today due to the works of François Édouard Anatole Lucas (April 4, 1842 - October 3, 1891, the inventor of the Tower of Hanoi and the Lucas Sequence) and Jacques Philippe Marie Binet (February 2, 1786 - May 12, 1856, the inventor of the rule for multiplying matrices) who studied the Fibonacci Sequence and found a formula to derive the n^{th} term of the Fibonacci Sequence. This formula was derived by Binet in 1843, and popularized by Lucas in his own work on the Lucas Sequence and also naming the Fibonacci Sequence, even though the result was known to Euler , Daniel Bernoulli, and de Moivre more than a century earlier (Knott, 2001).

Figure 1.1 Leonardo Pisano

1.2 BACKGROUND OF RESEARCH

1.2.1 The Fibonacci Sequence

The Fibonacci Numbers first appeared, with the name *maatraameru* (mountain of cadence), in the work of the Sanskrit grammarian, Pingala (*Chhandah-shāstra*, the Art of Prosody, 450 or 200 BC). Another Indian mathematician, Virahanka, gave particular conditions for the Fibonacci sequence in the 8th century. The Indian Jain philosopher, Hemachandra, revisited the problem in the 1100's. Sanskrit vowel sound's can be long (L) or short (S), and Hemachandra wished to compute how many cadences (A cadence is a particular series of intervals or chords that ends a phrase, section, or piece of music. Cadences give phrases a distinctive ending that can, for example, indicate to the listener whether the piece is to be continued or concluded.) of a given overall length can be composed of these long and short vowels. Hemachandra showed that the number of patterns of length, n, is the sum of the two previous numbers in the series thus relating his findings to the modern day Fibonacci Sequence. Another Indian scholar, Gopala (1135) had also done his work on the cadences using the basic principles of the modern day Fibonacci Sequence. As you can see, most of the relevant works around the Fibonacci Sequence earlier were concentrated in the field of language (Mathworld, 2003).

1.2.2 The Golden Section

Two numbers deduced from the Fibonacci Sequence, Phi, Φ and phi, φ are known as the Golden Section. The Golden Section was first discovered by ancient Greek mathematicians because of its frequent appearance in geometry and architecture. These figures also appeared in Sumerian tablets as early as 3200 BC, but there is no evidence that the Sumerians recognized the ratio. The Greeks usually attributes the discovery of the Golden Section to Pythagoras or to the Pythagoreans, notably Theodorus or Hippasus. Euclid in his book *Elements* gives the first known written definition of the Golden Section which he called, in English, "extreme and mean ratio". Before Euclid, Phidias (490–430 BC) made the Parthenon statues that seem to follow the Golden Ratio and Plato (427–347 BC) in his book, Timaeus, describes five possible regular solids, some of which are related to the Golden Ratio.

The Fibonacci Sequence, the Fibonacci Numbers and the Golden Section appear frequently in many different aspects of mathematics and science including nature, physics, architecture and astronomy. It is quite amazing that Fibonacci Number patterns occur so frequently in nature such as on flowers, shells, plants, leaves, galaxies and many more that this phenomenon appears to be one of the principal "laws of nature". Many plants show the Fibonacci Numbers in the arrangement of the leaves around the stem. Some pine cones and fir cones also show the Fibonacci Numbers in their whirls, as do daisies and sunflowers show the Fibonacci Numbers in their petals and seed head swirls. It is a belief that almost all living beings are governed by Fibonacci's work (Fibonacci Association, 2006).

1.3 THE GOLDEN RECTANGLE.

The Fibonacci Sequence, as introduced earlier in this dissertation, is a sequence of where the following number is the sum of its two previous numbers. Therefore the general formula of the Fibonacci Sequence is:

Define : $F_0 = 0$

$F_1 = F_2 = 1$

And : $$F_{n+1} = F_n + F_{n-1} \tag{1.1}$$

From this general formula of the Fibonacci Sequence, we get a sequence of numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610...

The Golden rectangle is built using the Fibonacci Numbers as the sides of squares that are drawn in a spiral. It's like drawing an anti-clockwise spiral of squares. This is how we draw a Golden Rectangle:-

Step 1:

Begin by drawing a 1unit x 1 unit square.

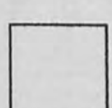

Figure 1.2 Step 1 drawing a 1 unit x 1 unit box

Step 2:

To its left, draw another one 1 unit x 1 unit square.

Figure 1.3 Step 2 drawing a 1 unit x 1 unit box

Step 3:

- Then above these squares draw a 2 unit x 2 unit square. An important rule is that the squares share their sides.

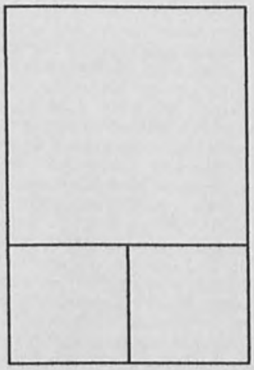

Figure 1.4 Step 3 drawing a 2 unit x 2 unit box

Step 4:

A 3 unit x 3 unit square is drawn to the right of these squares.

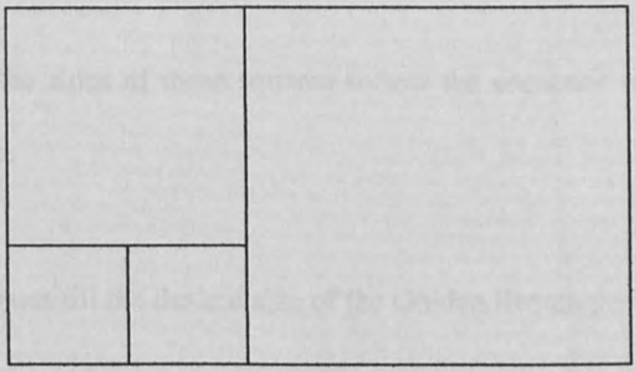

Figure 1.5 Step 4 drawing a 3 unit x 3 unit box

Step 5:

- Then draw a 5 unit x 5 unit square at the bottom of the previous squares.

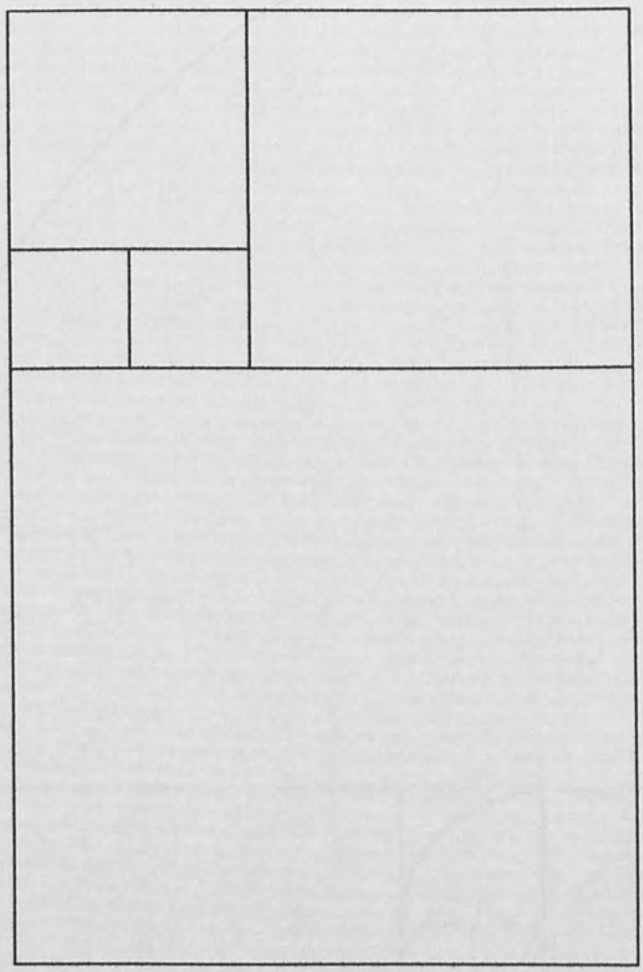

Figure 1.6 Step 5 drawing a 5 unit x 5 unit box

Step 6:

- As can be seen, the sides of these squares follow the sequence of numbers as the Fibonacci Sequence.

- The process continues till the desired size of the Golden Rectangle is obtained .

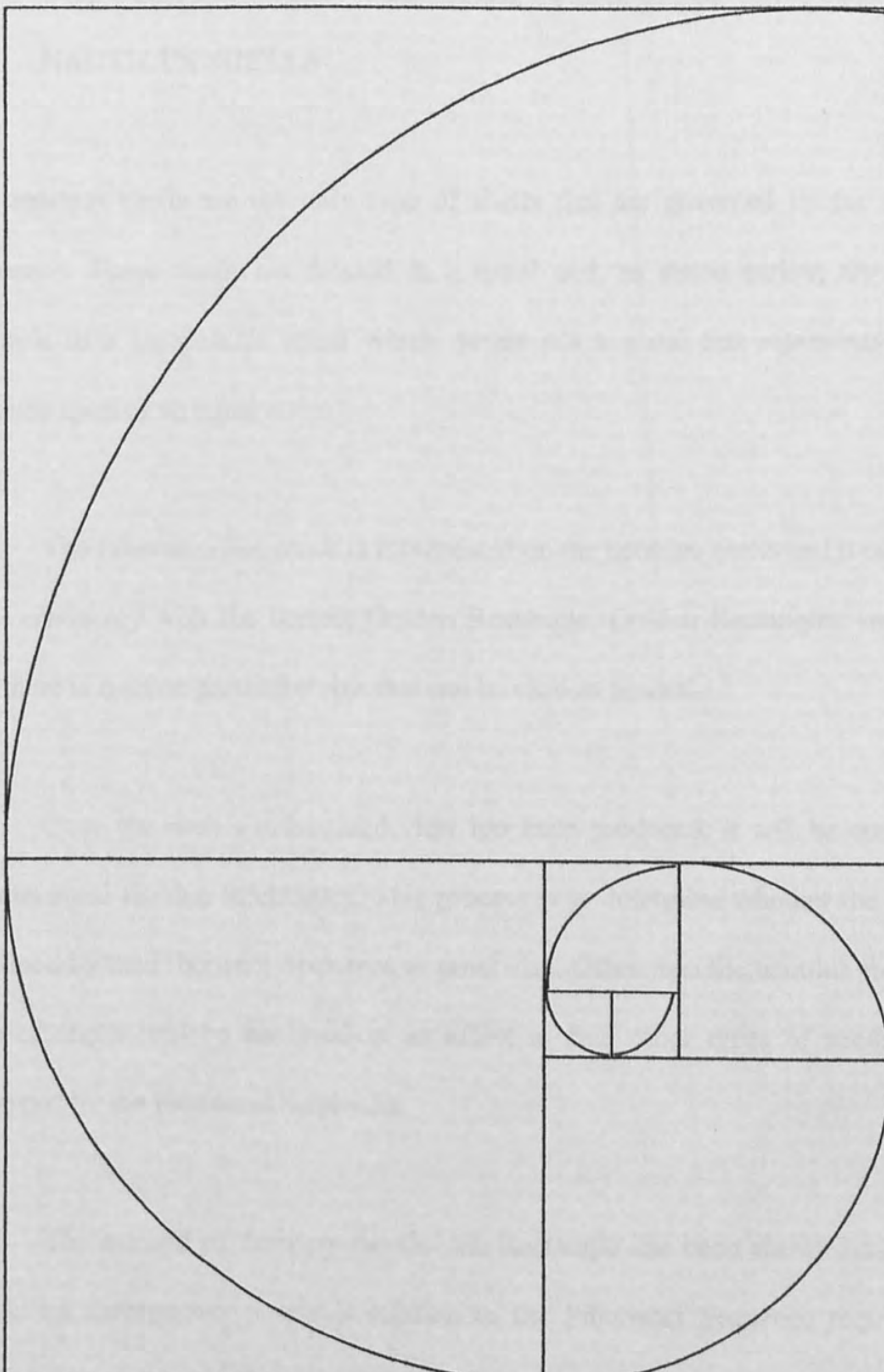

Figure 1.7 The basic plan of a spiral that is governed by the Fibonacci Sequence.

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