

Performance Analysis on Beam-steering Algorithm for Parametric Array Loudspeaker Application

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Abstract — A highly directional audible sound can be generated based on the nonlinear interaction of the ultrasonic sound wave in air. The direction of this audible sound beam is controllable by utilizing array signal processing technique for parametric array. However, most of the existing work done is focused on the algorithm improvement of the steering angle or reduce the computational intensity, all of which does not consider the signal to noise ratio (SNR) of the system. Low SNR cases will cause clipping, distortion or even signal loss towards the audible sound generated by the parametric loudspeaker. In this paper, simulation and performance analysis is carried out to demonstrate the signal noise ratio for different weighting functions in the beam-steering algorithm of the parametric array loudspeaker.

Keywords – parametric loudspeaker; array signal processing; directivity; beam-steering; signal noise ratio

I. INTRODUCTION

Parametric Array has been widely used in underwater sonar application due to its high directivity response. The nonlinear effect generated by high-level ultrasound was presented by Westervelt in 1960's [1]. Westervelt found that if two high frequency beam of sound collimated in the same direction, it will produce a difference frequency signal. The propagation of the resultant sound beam characterizes its high directivity.

In 1965, Berklay invented the theory of amplitude modulation [2]. He shows that the difference frequency can be obtained by modulating the ultrasonic carrier frequency with its primary wave. Later, Bennett and Blackstock had successfully carried out experiment to show that the parametric array was realizable in air [3]. Due to the availability of advanced signal processing methods and the development of high power transducer, it materialized the possibility of parametric array for acoustic application.

With the realization of the nonlinearity phenomena of sound beam, Yoneyama and Fujimoto constructed the first novel directional parametric loudspeaker design in 1983 [4]. Their experiment showed that by modulating the amplitude of the ultrasonic carrier through the ultrasonic transducer array,

they were able to generate a “self-demodulation” broadband signal with high directivity.

Attention and interest on the area had rapidly increased since then. Most of the efforts [5,6,7] was put to pre-processing scheme to reduce unwanted harmonic distortion. Modelling of the parametric array loudspeaker [8,9] had been done to demonstrate the nonlinear sound beam generated by the parametric loudspeaker. A new kind of beam-steering algorithm for difference frequency was developed by Gan *et al.* [10]. These studies achieved a notable improvement in signal processing. However, not much work explained the effect of the signal to noise ratio for different weighting functions in beam-steering algorithms. In this paper, different kinds of beam-steering technique in uniform linear array as show by Orfanidis [11] had been carried out to explain the behavior of SNR.

This paper is organized as follows: In section II, the theory of the parametric array is presented. In section III, the SNR model for beam-steering base on parametric array is explained. Various array beam-steering methods are presented in section IV and section V contains the simulation results. Lastly, the conclusion will be made in section VI.

II. THEORY OF PARAMETRIC ARRAY

When two high-level ultrasonic waves f_1 and f_2 are collinearly emitted, a sum frequency of $f_1 + f_2$ and a difference frequency $f_1 - f_2$ will appear due to the interaction with the medium. Due to this nonlinear interaction, the absorption coefficient is proportional to the frequency squared. Therefore, high frequency terms $2f_1$, $2f_2$ and $f_1 + f_2$ and other higher harmonics will decay rapidly as the distance increases from the parametric array loudspeaker. After a short distance of wave propagation, only the low frequency term which is difference frequency $f_1 - f_2$ with sufficient amplitude will remain within human audible range. Fig. 1 illustrates the nonlinear interaction process of the parametric array and Fig. 2 shows the sound beam production of the nonlinear interaction.

The signal model of the collimated wave is defined as (1).

$$p_1(t) = P_1 E(t) \sin(\omega_c) \quad (1)$$

where P_1 is the amplitude of the signal, $E(t)$ is the modulation envelop and ω_c is the carrier angular frequency. Equation (1) will demodulate after the nonlinear interaction. The wave pressure can be explained by (2).

$$p_s(t) = \frac{\beta P_1 a}{16\pi\rho_0 c_0^4 z \alpha} \frac{\partial^2}{\partial t^2} E^2(\tau) \quad (2)$$

where $\beta = (\gamma + 1)$ is the coefficient of nonlinearity ($\beta_{air}=1.2$), γ is the ratio of specific heats, a is the transducer radiating area, ρ_0 is the density of air, c_0 is the small-signal wave propagation speed, z is the axial distance, α is the absorption coefficient of air for the carrier frequency. Expression (2) describes that the demodulation signal is dependent to the modulation envelope of the signal. Although many methods of pre-preprocessing scheme can apply based on equation (2), but the interest of this paper does not lie on the pre-processing methods. Therefore, conventional amplitude modulation will be used as the signal model for analysis in the further sections. The conventional AM is described in (3).

$$E(t) = 1 + mg(t) \quad (3)$$

where m is the modulation index of the signal, $g(t)$ is the signal of interest which can normally assume as a normal periodic signal with a certain frequency.

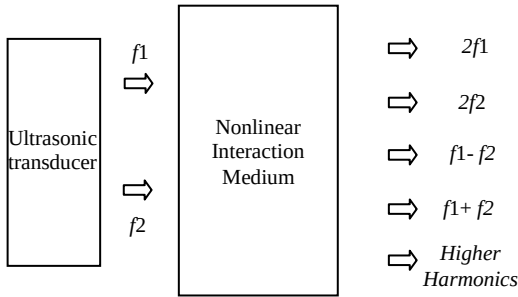


Figure 1. Nonlinear interaction process.

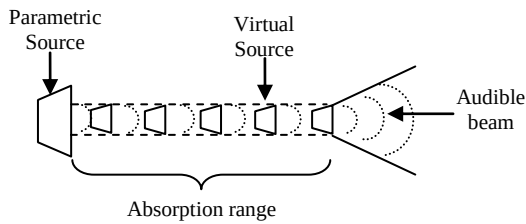


Figure 2. Sound beam production of parametric array.

III. SNR MODEL FOR BEAM-STEERING

SNR is a measure of the signal level to the noise level. To measure the SNR, the geometric arrangement could inference the value of SNR. Fig. 3 geometric arrangement of uniform linear array could be implemented on parametric array loudspeaker design.

Consider the observation signal generated by the uniform linear array beam-steering as (4),

$$y(k) = w^H a(\theta_s)_i s(k) + w^H w(k) \quad (4)$$

where $w = [w_1, w_2, w_3, \dots, w_N]$ is the weight vector for the beam-steering, $(\cdot)^H$ denote the hermitian of w , N is the number of transducer, $a(\theta_s)_i = [\exp(-j(2\pi/\lambda)d(i-1)\sin(\theta_s))]$ is the steering vector of the array, d is the distance between two transducers, λ is wavelength of the carrier frequency, $s(k)$ is the signal of interest as described in section II, and $w(k)$ is thermal transducer noise which can represented by a Gaussian noise with zero mean and unit variance. The resulting SNR of linear array beam-steering output array can be described in equation (5).

$$SNR_{array} = \frac{|w^H a(\theta_s)|^2 \sigma_s^2}{\|w\|^2 \sigma_w^2} \quad (5)$$

where $\sigma_s^2 = E\{|s(k)|^2\}$, $\sigma_w^2 = \{ |n(k)|^2 \}$, $\|w\|$ is signal power, noise power of a single element and the L2-norm of the weight vector.

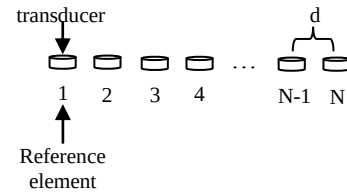


Figure 3. Uniform linear array.

IV. ARRAY BEAM-STEERING METHOD

The array beam-steering design for the parametric loudspeaker application must have narrow-beam and low-sidelobe characteristic. The reason of the narrow-beam and low-sidelobe is to control the directivity while reduce the noise that due to thermal transducer of the electronic component. Therefore, the design method choice for analysis will be based on this criterion. There are four types of narrow-beam and low-sidelobe weighting designs were choose. They are uniform, Dolph-Chebyshev, Taylor, and Prolate weighting design. Fig. 4 to Fig. 6 shows the polar plot of the beam

TABLE I. WEIGHTING METHOD AND 3dB BEAMWIDTH

Method	Weighting function	3dB-Beamwidth
Uniform	$W=[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$	8.91 °
Dolph-Chebyshev	$W=[0.0823, 0.1715, 0.2908, 0.4010, 0.4674, 0.4674, 0.4010, 0.2908, 0.1715, 0.0823]$	12.1 °
Taylor	$W=[0.0258, 0.1117, 0.2460, 0.3975, 0.5181, 0.5181, 0.3975, 0.2460, 0.1117, 0.0258]$	13.0 °
Prolate	$W=[0.0609, 0.1583, 0.2842, 0.4038, 0.4769, 0.4769, 0.4037, 0.2842, 0.1583, 0.0608]$	12.8 °

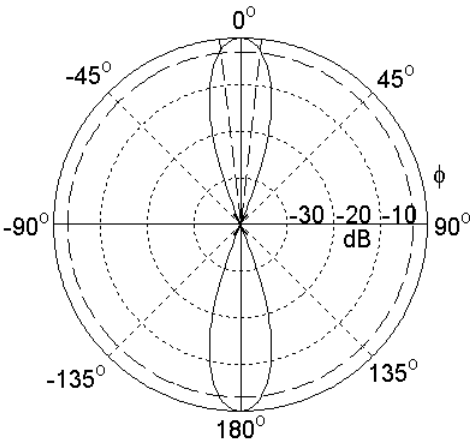


Figure 5. Polar plot using Taylor weighting design (10 element and 90°).

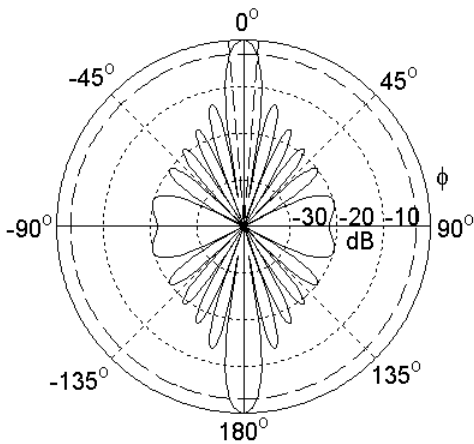


Figure 4. Polar plot using uniform weighting design (10 element and 90°).

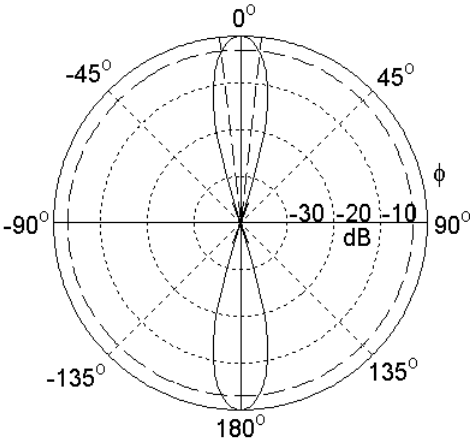


Figure 6. Polar plot using Prolate weighting design (10 element and 90°).

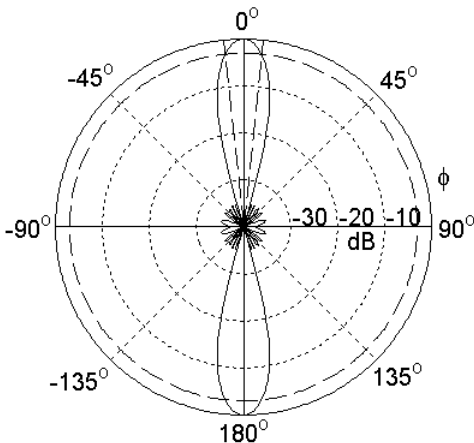


Figure 5. Polar plot using Dolph- Chebyshev weighting design (10 element and 90°).

V. SIMULATION RESULTS AND DISCUSSION

For the simulation, the carrier frequency of the modulation is set to 40 kHz. The sampling frequency is set to 160 kHz and the output signal frequency is set to 1 kHz of sine wave. The speed of the sound is assumed as 344 ms⁻¹. The inter element spacing is 49 mm. For Dolph-Chebyshev, Taylor and Prolate weighting function design, the sidelobe will be set as 40 dB. The number of the transducer element was set from 0 to 200. The simulation was then run with respective beam-steering angles of 0°, 30°, and 60° to compare the SNR output of each type of the beam-steering design. The result was show in Fig. 7 to Fig. 9.

The effect of SNR value by increase number of transducer has show in Fig. 7. During steering angle of 0°, increasing number of transducer will cause an exponential increase of SNR value. SNR value for Uniform method is much more prominent compare to other method. However, SNR value of

all the method tends to increase logarithmically after 75 transducers. This means that by further increase the number of transducer will not have significant improvement on SNR value. The uniform SNR value has the highest SNR value follow by Dolph-Chebyshev, Prolate and Taylor method.

Similar result in Fig. 7 could not be found in Fig. 8. When the steering angle is 30 °, all of the methods experience common notches on every increase of approximately 76 transducers. Prolate and Taylor method are slightly different from the others because both method experience first notch on the second notch of uniform and Dolph-Chebyshev method. Uniform and Dolph-Chebyshev method have approximately 80 transducers per ripple peak interval. For uniform method, each ripple peak experience slight decrease with approximately rate 2.4 dB per transducer. This shows that if the steering angle is not 0 °, even a larger transducer number would not provide improvement in SNR value. the same happens to Taylor and Prolate method. For Dolph-Chebyshev case, it experiences an inverse result from uniform method. Each ripple peak has improvements in SNR value. However, SNR value still much lower compare to other method. To achieve similar SNR value as other method, this requires increasing transducer number to a very large value.

More ripple effects of SNR value was found in Fig. 9. when the steering angle is set on 60 °, all of the method tend to form similar ripple pattern for all of the method. Uniform, Prolate and Taylor method experience dramatically drop in SNR value before the number transducer of 20. Onward, the SNR value tends to attenuate constantly for uniform, Prolate and Taylor method. The Dolph-Chebyshev method has approximately same SNR value as uniform method when the transducer number is increased to 89 and leading in SNR value after that point. Uniform method show two sudden drops in SNR value to -718.9 dB and -689.8 dB at number transducer 86 and 172 respectively. This shows the SNR value could be worst on certain transducer number. Avoid choosing those particular transducer numbers is a very good idea to design the parametric array loudspeaker.

To further demonstrate the change in SNR value with different steering angle, the number of transducer is set to 10, 20, 40 and 80. The simulation is then run with angle ranging from 0 ° to 90 ° to observe SNR output of each type of the beam-steering design. The result is show in Fig. 10 to Fig. 13.

The effect of SNR value by increase steering angle has show in Fig. 10. Uniform method has the highest SNR value as expected from the previous result follow by Dolph-Chebyshev, Prolate and lastly Taylor method. All of the method tends to form very similar patterns. However, Taylor method only could form similar pattern as other method after 48 °. All of the methods has approximately 6 ripples except for Taylor method. Each width between two ripple peak is approximately 12 °.

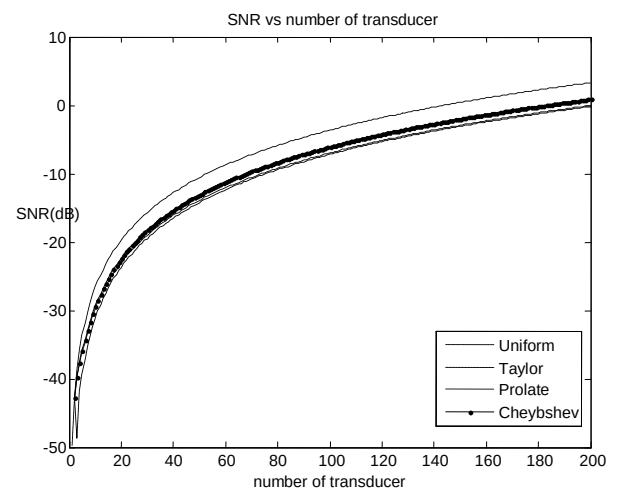


Figure 7. SNR vs number of transducer for 0° beam-steering.

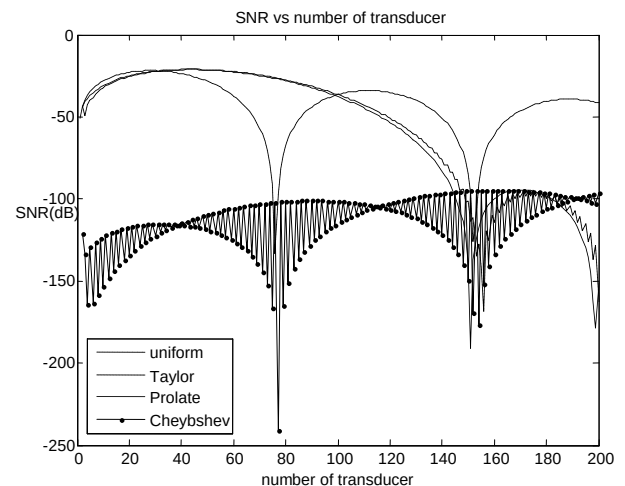


Figure 8. SNR vs number of transducer for 30° beam-steering.

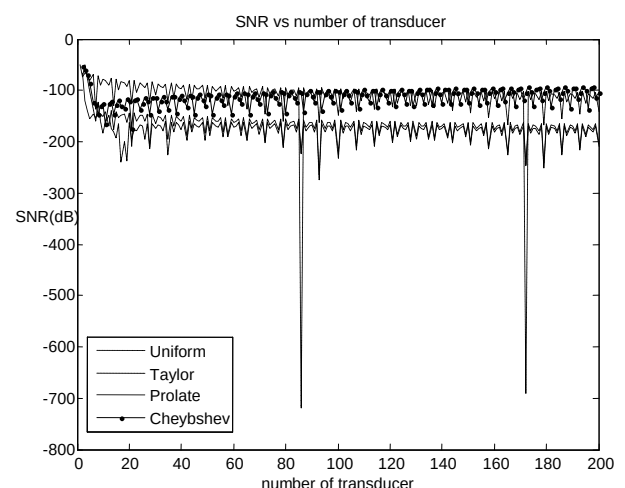


Figure 9. SNR vs number of transducer for 60° beam-steering.

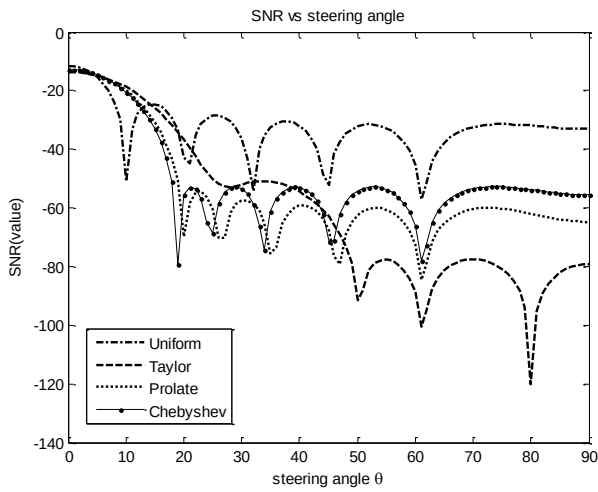


Figure 10. SNR vs steering angle of 10 element transducers.

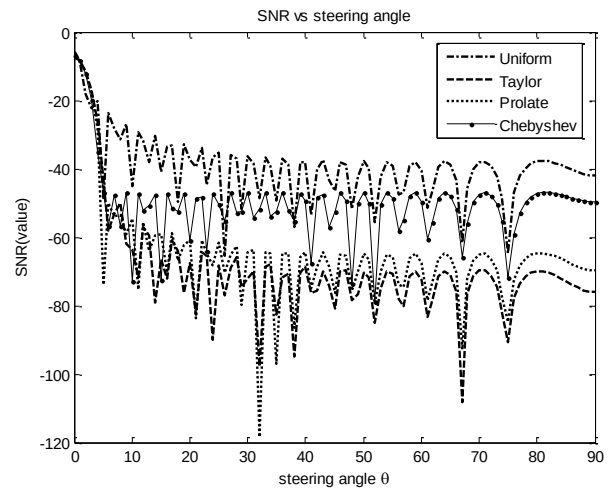


Figure 12. SNR vs steering angle of 40 element transducers.

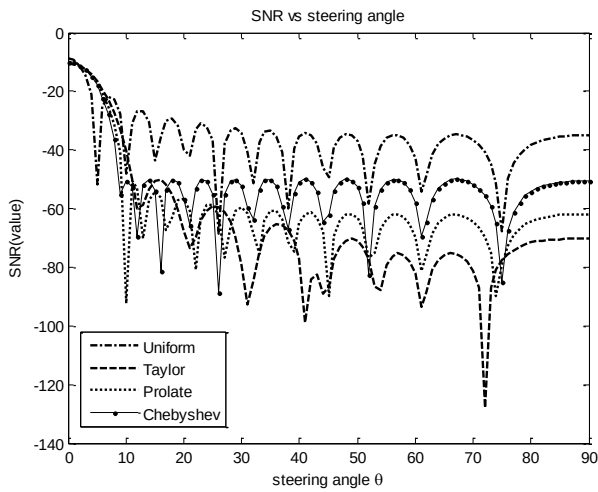


Figure 11. SNR vs steering angle of 20 element transducers.

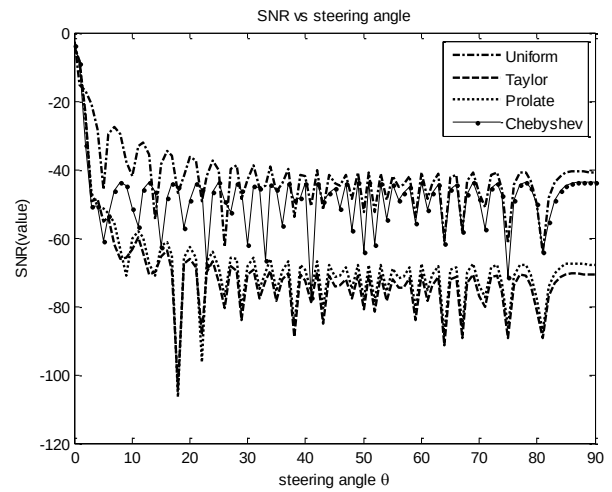


Figure 13. SNR vs steering angle of 80 element transducers.

In Fig. 11, Fig. 12 and Fig. 13, notice that the total number of ripple has increased twice when the transducer number increase twice. This shows that when increasing number of transducer will linearly increase the number of ripple in the beam-steering range from 0° to 90° . Each width between two ripple peaks also reduces half as increasing twice the transducer number. The average ripple peak between two ripple is by 5° , 2.5° , and 1.25° for Fig. 13, Fig. 14 and Fig. 15 respectively. Taylor method could form similar pattern as other method at lower steering angle when increase the transducer number. It form similar pattern after angle of 25° , 12.5° , 6.3° for Fig. 11, Fig. 12 and Fig. 13 respectively.

There are some general form of relationship can be obtained from the result. The width between two ripple peaks will reduce linearly when increasing number of transducer. This will increase the probability of getting maximum SNR value. As a trade off of it, the probability of getting notch SNR

is also higher. Therefore, parametric array loudspeaker with large number transducer is easier to optimize due to the width between ripple is very small. Larger transducer number experience dramatically drops in SNR value when the beam-steering angle increases. When increasing the transducer number, the average SNR value for Dolph-Chebyshev, Taylor and Prolate method remains approximately same in the angle range from 0° to 90° . However, average SNR value for uniform method experience drop when transducer number is increased.

VI. CONCLUSION

The results show that the SNR value exhibits differently with the change of beam-steering angle and number of transducers. The SNR value will affect the sound signal generate by the parametric array. Therefore, optimum weighting function should be selected in order to obtain a

good tradeoff between SNR value and other design criteria like sidelobe level, possible angle of beam-steering without grating lobe, and also the total gain of the output. For future study, it will be more interesting to include hardware amplifier of the parametric array loudspeaker system for more complete analysis on the output signal SNR. Analysis of the SNR value for different type of geometrical arrangement might exhibit different results too.

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