4 x 4 SUDOKU MADE FOR CHILDREN

SUSANNA TOH SIEW TIE

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Alamat Tetap: 834B, Taman Bintulu,
Jalan Tanjung Batu,
97000 Bintulu, Sarawak

Tarikh: 18/04/2007

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   (Mr. Rajasegeran s/o Ramasamy) _____

2. **EXAMINER**
   (Ms. Khadizah Ghazali) _____

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ABSTRACT

This study is conducted on 4 x 4 Sudoku puzzle designed special for children. A 4 x 4 Sudoku puzzle is contain with 4 x 4 grids that divided into four 2 x 2 regions. The goal is of this game is to complete the puzzle by filling it with the colour {Red, Green, Blue, Yellow} in every cell in each row, column, and region exactly once. Games-in-Education help brain development in the younger age. The concentration of colour is highlighted to attract children to play the game. All of the 4 x 4 Sudoku puzzle's theory discussed in this paper is obtained from the 9 x 9 Sudoku. There are two methods for solving, namely hand-solved method and computer-solved method. There are different techniques used for the hand-solved puzzle such as "Unique Missing Candidate", "Locked Candidate", "Hidden Single" and so on. While computer-solved puzzles are solved by using a C based computer program. A total of 288 possible solutions can be obtained from the 4 x 4 Sudoku puzzle.

ABSTRAK

Kajian ini telah dijalankan ke atas 4 x 4 Sudoku yang khas untuk kanak-kanak. 4 x 4 Sudoku adalah terdiri daripada 4 x 4 grid yang dibahagikan kepada empat 2x2 bahagian. Matlamat permainan ini adalah untuk mengisikan setiap kotak dengan menggunakan warna {Merah, Hijau, Biru, Kuning} dalam setiap baris, lajur, dan bahagian sebanyak satu kali. Kepentingan tentang kajian ini adalah untuk mencari penyelesaian yang efektif dengan penggunaan warna. Warna boleh merangsang pemikiran kanak-kanak dan menarik perhatian mereka. Pendedahan kepada 4 x 4 Sudoku pada peringkat awal umur kanak-kanak memberi manfaat terhadap pembinaan pemikiran kanak-kanak. Konsep dan idea daripada 4 x 4 Sudoku ini diperolehi daripada 9 x 9 Sudoku. Selain itu, 4 x 4 Sudoku boleh diselesaikan secara manual atau dengan menggunakan komputer. Kaedah manual menggunakan pelbagai teknik yang berbeza iaitu "Unique Missing Candidate, "Locked Candidate", "Hidden Single", dan sebagainya. Manakala kaedah komputer pula menggunakan pengaturcaraan C untuk mencari penyelesaian. Cara untuk mencari 288 penyelesaian akhir daripada permainan ini boleh didapati daripada 4 x 4 Sudoku ini. Secara keseluruhan, pengaturcaraan C merupakan satu kaedah yand ideal untuk menyelesaikan 4 x 4 Sudoku.

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CHAPTER 1

BACKGROUND

1.1 Introduction

Sudoku, also known as number place is a logic-based placement puzzle (Wikipedia, 2005). The standard puzzle of Sudoku contains 81 cells and presented on a square grid that is usually 9 x 9 grids that divided into nine 3 x 3 regions (also called as blocks, sub grids or boxes) (Felgenhauer & Jarvis, 2005, 2006). Figure 1.1 is an example of 9 x 9 Sudoku puzzle.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
A	9	5	3	2	1	4	7	6	8
B	2	7	6	8	5	3	4	1	9
C	8	1	4	6	7	9	2	3	5
D	7	4	8	5	3	1	6	9	2
E	6	9	1	7	4	2	5	8	3
F	5	3	2	9	6	8	1	7	4
G	1	6	9	4	8	5	3	2	7
H	3	2	5	1	9	7	8	4	6
I	4	8	7	3	2	6	9	5	1

Figure 1.1 Example of solved 9 x 9 Sudoku puzzle

The goal is to complete the grid by fill in the empty cells with only one of the numerical digit one to nine in each row, column, and region. It is a simple concept but can be complex to solve.

The name Sudoku is come from Japan. At first it is named in a longer phrase as "Suuji wa dokushin ni kagiru", translated as "the digit must remain single". It means "single number" because just one number can appears in each cell. Later, the name was abbreviated as Sudoku for easier pronunciation. It was in Japan that the puzzle first becomes popular (Davis, 2006; Wikipedia, 2005).

The initial set of Sudoku always comes with a started grid where some cells are already contains numbers. The cells are known as "givens" (Santos-Garcia & Palomino, 2006). Sudoku is not a mathematical puzzle, so there is no counting work when playing Sudoku (Hayes, 2006; Santos-Garcia & Palomino, 2006). The only requirement when completing the puzzle is to be patience and always think logically.

When something is called as a "puzzle", it is a game with unique solution (Cazenave, 2006). Similarly, it should be noted that there is only one valid solution for each puzzle. However, different set of initial puzzles can have same solution (Taalman, 2007). Figure 1.3 shows that an example of solved Sudoku puzzle (Figure 1.2) can have different set of initial puzzles.

	1	2	3	4	5	6	7	8	9
A			3	2					
B		7	6				4	1	
C		1	4			9			5
D			8		3				2
E				7	4	2			
F	5				6		1		
G	1			4			3	2	
H		2	5				8	4	
I							9		

	1	2	3	4	5	6	7	8	9
A				2				6	8
B	2	7		8				1	9
C			4						
D		4		5	3				
E		9						8	
F					6	8		7	
G							3		
H	3	2				7		4	6
I	4	8				6			

Figure 1.2 Two Sudoku puzzles with the same solution

As a matter of fact, the symbol in the grid is not necessary to be numbers at all. The use of numbers in Sudoku is for players' convenience. The numbers can be replaced with letters, pictures, shapes, and some other symbols. In some cases, there are even mixture of letters and digits used to increase the difficulty of the puzzle (Hayes, 2006; Santos-Garcia & Palomino, 2006).

For any positive integer n, an order-n Sudoku grid with n^2 rows, n^2 columns and n^2 regions can be drawn. It means that the grid has a total of n^4 cells, which are to be filled with numbers in the range from one to n^2 (Frank, 2005; Hayes, 2006) For instance, if put the n to be four, then it will be an order-4 Sudoku with 16 rows, 16 columns, 16 regions and 256 cells in total.

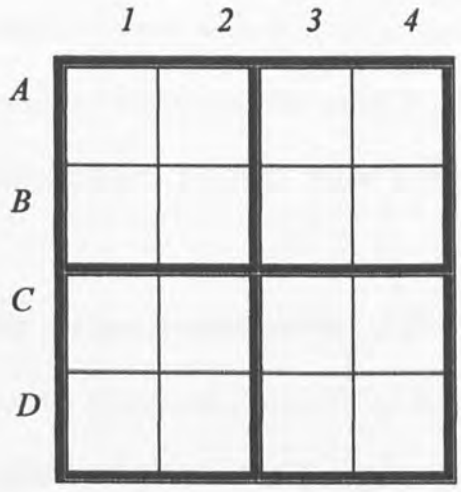

Figure 1.3 4 x 4 grids Sudoku

This dissertation will discuss about an order-2 Sudoku, which is the 4 x 4 Sudoku. The puzzle is presented in 4 x 4 grids which divided into four 2 x 2 regions and with the total of 16 cells as shown in Figure 1.3.

The 4 x 4 Sudoku is also known as Shi Doku by the Japanese (means "four singles") or mini-Sudoku (Arcos *et al.*, 2006; Taalman, 2007). It is a 4 x 4 grids that divided into four 2 x 2 regions, look alike with 9 x 9 Sudoku and has the same theory with the original puzzle. Therefore, the theory in 9 x 9 Sudoku can be used and apply it in the 4 x 4 Sudoku. The goal now is to complete the puzzle by fill in the numbers one to four into the remaining cells. Each row, column and region of the puzzle must contain one of the numbers one to four exactly once.

The main idea of this paper is to produce 4 x 4 Sudoku made for children. The use of number will be eliminated and replace by using colour as the symbol of solving the 4 x 4 Sudoku. The purpose of using colour as the symbol of the puzzle is to make the puzzle more interesting and also to attract children playing the puzzle.

The 4 x 4 Sudoku puzzles are no challenge at all for the adults, but it might be challenging for the children. Moursund (2006) stated that games can be an important of both informal and formal education. A puzzle or sometimes called brain teaser is problem designed to challenge one's brain and to be entertaining. Doing a puzzle is like doing a certain type of brain exercise, which brings benefits for the players. Much of the children's learning occurs during playing games.

Besides, playing games also helps brain development in the younger ages. When solving Sudoku puzzle, it requires logical thinking. Therefore, the 4 x 4 Sudoku can be used as one of the games type to be the tool in the education field. This idea may be proposed for the early child learning in the education field in Malaysia and help our younger generation to gain more intelligent.

1.2 How to Sudoku?

When playing Sudoku, the empty cells cannot be filled in anyhow or willful. There is one rule that must be obeyed during the solving process, which is the *Rule of One* (Arcos *et al.*, 2006). This rule requires a single number (from one to four) to be entered in each blank cell so that each number is used only once in each region and only once in each row and column in the 4 x 4 grid.

The concept of Sudoku is very simple. There are no calculations needed in solving the puzzle. In other words, there is no mathematics or arithmetic is required. There is no multiplication, subtraction or any other operations needed between rows and columns to get the answer (Hayes, 2006; Santos-Garcia & Palomino, 2006). The only skill to complete the puzzle is to think logically. Sometimes when cannot get any clue to continue the puzzle, the guessing skill can also be used by simply choose a candidate from a few possible candidates. It may lead to the final answer but this skill easily causes errors or contradictions and it is hard to detect once there is a mistake.

The Sudoku puzzle can either been hand-solved or computer-solved. Most people like to solve the puzzle by hand to experience the fun of the puzzle rather than

solving using computer. That is why the Sudoku is also known as pencil-and-paper puzzle (Arcos *et al.*, 2006). The common process of solving is involving scanning, marking up and analyzing, which is finding the possible candidates in each cell (Santos-Garcia & Palomino, 2006). This technique make the puzzle look easier, as in every cell it has its own possible candidates and make the determination of the answer easily and faster.

1.3 Research Objective

The objective of this research is to:

i. Create 4 x 4 Sudoku puzzle made for children using colour.

ii. Show techniques to solve 4 x 4 Sudoku puzzle.

iii. Find the number of possible solutions for 4 x 4 Sudoku.

iv. Find the minimal clues in the initial set of the puzzles to get a unique solution.

v. Solve the 4 x 4 Sudoku manually and using computer.

vi. Study the theory of 9 x 9 Sudoku and apply it in 4 x 4 Sudoku.

1.4 Research Scope

The scope of this research is:

i. Limited 4 x 4 Sudoku puzzle.

ii. Using C programming language to create a 4 x 4 Sudoku program.

CHAPTER 2

LITERATURE REVIEW

2.1 History of Sudoku

Although "Sudoku" is a Japanese name, it is not origin from Japan (Felgenhauer & Jarvis, 2006). The puzzle is actually an invention of European and American. It is claims that the puzzle was invented by Leonhard Euler, a Swiss mathematician who made enormous contributions to a wide range of mathematics and physics field in the 18^{th} century. Euler was said to have invented variations in calculus, functions in geometry and many more equations and ideas (Wikipedia, 2005).

However, this is not exactly the truth because Euler had only created the basic of "Sudoku", which is the concept of Latin Squares. There is another person who invented Sudoku in a later date. Euler apparently developed the concept of Latin Squares into a square where numbers in a grid appear only once, across up and down. The idea on Latin Square was then adapted into Sudoku (Wikipedia, 2005).

Sudoku puzzles are actually special case of Latin Squares, where any solution of Sudoku puzzles must also be the solution of Latin Squares, but not every solution of Latin Squares can be the solution of Sudoku puzzles (Felgenhauer & Jarvis, 2005,

2006). Latin Square is an n x n multiplication table. It can be treated as a square matrix such that each row and each column contain n different element (Cazenave, 2006).

In 1979, Howard Garns, a retired architect and freelance puzzle constructor invented the modern puzzle Sudoku in the age of 74 by using Euler's concept and applied it into a 9 x 9 square grid with addition nine 3 x 3 sub-grids (Frank, 2005; Hayes, 2006; Santos-Garcia & Palomino, 2006).

Garns then contributed the puzzle to the puzzle publisher Dell Magazines. The puzzle is named as "number place" by Dell. The puzzle then published in New York for the first time in a puzzle magazine *Math Puzzles and Logic Problems* (Davis, 2006; Frank, 2005; Hayes, 2006; Santos-Garcia & Palomino, 2006).

The puzzle then brought to Japan by Nikoli. It was first introduces in Japan in *Monthly Nikolist Magazine* in April 1984 (Hayes, 2006; Santos-Garcia & Palomino, 2006). The president of Nikoli, Koji Maki named the puzzle as "Suuji wa dokushin ni kagiru", meaning that "the digit must remain single" or "the numbers must occur only once". However, it is too long to pronounce and at a later date, the name was abbreviated to Sudoku (pronounced as sue-do-ku; su=number, doku=single) (Davis, 2006; Hayes, 2006; Santos-Garcia & Palomino, 2006; Wikipedia, 2005).

Sudoku start to become popular and was surged from a revival at Japan. Later, different versions of Sudoku have been created and putting the game in new twists.

There are giant Sudoku puzzles introduced such as 16 x 16 and even 25 x 25 squares (Davis, 2006).

In 1997, Wayne Gould, a retired New Zealand High Court Judge discovered Sudoku on a trip to Japan. He spent a few years to write a computer program to generate the puzzles. Gould then persuaded and convinced *The Times* to print out the puzzle. In November 2004, his puzzles were first appeared in four levels of difficulty (easy, mild, difficult, and fiendish). The phenomena of popularize led the others publisher to launched the puzzle in their newspapers. This eventually led to the puzzle coming back to the United States with growing popularity (Hayes, 2006).

2.2 Literature Reviews

There are some literatures on Sudoku that have been done by different mathematician which is related with this dissertation.

2.2.1 Tom Davis

Tom Davis wrote a research paper about Sudoku with the title *The Mathematics of Sudoku*. Davis has stated that the 9 x 9 Sudoku can be extended to a larger puzzle and also gives some fairly standardized terminology for 9 x 9 Sudoku that can be used to apply it in the 4 x 4 Sudoku.

Davis had state that a "square" refers to one of the 81 boxes in the Sudoku grid, each of which is to be filled eventually with a digit from 1 to 9. A "block" refers to a

3 x 3 sub-grid of the main puzzle in which all the numbers must appear exactly once in a solution. While a "candidate" is a number that could possibly go into a square in the grid.

A few techniques that obviously used to solve 9 x 9 Sudoku were shown in the paper. The strategies used to solve the puzzle are "Unique Missing Candidate", "Naked Single", "Hidden Single", "Locked Candidate", "Naked Pair", "Hidden Pair", "X-Wings" and so on.

There are some explanations given on each technique. The "Unique Missing Candidate" technique used when 8 of 9 elements in any virtual line (row, column, or region) are already filled in cells. Then the only empty cell must contain the element which is missing.

The "Naked Single" technique is used when there is only one possible candidate v that is valid in a cell S, and then the candidate v can be assigned into S. While the "Hidden Single" technique required to look in every virtual line (row, column or region) for a candidate which appears in only one of the cells in that virtual line. When this occurs, it means that a hidden single was found and can be assigned into the cell.

"Locked Candidates" used when a region where one of the possible candidates is in the same row or column within the region was found. Since the candidate must appear in the row or column in the region, the candidate can be eliminated from the row or column with other region.

The "Naked Pair, Triplet, Quad" technique requires a naked pair, triplet or quad to be in the same virtual line. Two values of a naked pair must be the only values that occur in two cells (similarly, a naked triplet refers to three values a naked quad refers to four values). When this occurs, the involving values can be eliminated as candidates from the other cells in the same virtual line.

"Hidden Pair, Triplet, Quad" technique are related to the use of naked pairs, triplets and quads. It is a technique about spotting matching groups of probabilities in a row, column or region. The involving candidates then can be discarded from the others cells in the same virtual line.

An "X-Wing" technique occurs when a candidate appears exactly twice in two rows and in the same column of these two rows. The cells where the X-Wing candidate can go form a rectangle. Therefore, a pair of opposite corner must contain the candidate. The connection of the pairs of candidate would form an 'X'. That is where it gets the name X-Wing. The "Swordfish" technique behaves alike with an X-Wing except that there must be three rows with the three candidates appear in three columns.

Lastly is the "Guessing" technique. This technique used when cannot solve the puzzle using the others techniques while there do exist a unique solution for the puzzle. It can be done by simply making a guess and examining the consequences of the guess. If that puzzle cannot be solved under the leads of the guess, then return to the starting point and discard the guess as a candidate from a certain cell.

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