# GENERATING AND VISUALIZING LATIN CUBE AND SUDOKU CUBE 

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#### Abstract

Latin Square and SuDoku is $n \times n$ array and can extend to three dimensions which can form a cube. This research is to determine whether different layers of Latin Square or SuDoku can form a cube or not, which is called Latin Cube and SuDoku Cube respectively. This research also is to investigate whether a Latin Cube or SuDoku Cube is unique or not. Row interchanging is used to find the matching layer for a Latin Square to form a cube. Method 1 and Method 2 are used to find the matching layer for a SuDoku to form a cube. The basic idea of Method 1 and Method 2 is from row interchanging and column interchanging. Two programs are written in $\mathrm{C}++$ computer language by using these methods, for Latin Cube and SuDoku Cube respectively. These programs can find the matching layer for a Latin Square and SuDoku, and save the outputs in VRML file type (.wrl). These outputs, or cube, can be viewed in a 3D environment by using VRML. The cubes can be viewed and studied in detail using VRML.


## PENGHASILAN DAN PEMBAYANGAN KUBUS LATIN DAN KUBUS SUDOKU


#### Abstract

ABSTRAK

Segiempat Latin dan SuDoku adalah tersusun dalam dua dimensi, dan boleh dilanjutkan ke tiga dimensi untuk membentuk satu kubus. Kajian ini adalah untuk menentukan samada segiempat Latin atau SuDoku yang tidak sama, boleh membentuk satu kubus atau tidak. Kubus ini masing-masing dipanggil Kubus Latin dan Kubus SuDoku. Kajian ini juga untuk menyelidik samada satu Kubus Latin atau Kubus SuDoku adalah unik atau tidak. Teknik penukaran baris digunakan untuk mencari lapisan yang sesuai bagi sesuatu Segiempat Latin supaya dapat membentuk satu kubus. Cara 1 dan Cara 2 digunakan untuk mencari lapisan yang sesuai bagi sesuatu SuDoku supaya dapat membentuk satu kubus. Idea asas bagi Cara 1 dan Cara 2 adalah dari teknik penukaran baris dan lajur. Dua program telah ditulis dalam satu bahasa komputer, iaitu $\mathrm{C}++$, dengan menggunakan cara-cara ini dan masing-masing untuk mencari Kubus Latin dan Kubus SuDoku. Program-program ini boleh mencari lapisan yang sesuai untuk sesuatu Segiempat Latin dan SuDoku, dan outputnya akan disimpan sebagai jenis fail untuk VRML (.wrl). Output-output atau kubus-kubus ini boleh dipandang dalam satu keadaan 3D dengan menggunakan VRML. Kubus-kubus dapat dipandang dan dikaji dengan lebih jelas di dalam VRML.


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## CHAPTER 1

## INTRODUCTION

### 1.1 BACKGROUND

A Latin Square in the order of $n$ is an $n \times n$ array where the entries are from a set of $n$ distinct symbols. These symbols are arranged in such a way that each symbol occurs exactly once in each row and in each column (Crownover \& Thibault, 2005). Figure 1.1 and Figure 1.2 show examples of Latin Square in order 3 and order 4.

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 2 | 3 | 1 |
| 3 | 1 | 2 |

Figure 1.1 Example of a Latin Square of order 3

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 2 | 3 |
| 3 | 4 | 1 | 2 |
| 2 | 3 | 4 | 1 |


| a | b | c | d |
| :---: | :---: | :---: | :---: |
| d | a | b | c |
| c | d | a | b |
| b | c | d | a |

Figure 1.2 Latin Square order 4 with different entries

SuDoku is an $n \times n$ array where $n$ is equal to $m^{2}, m$ is natural number. We can simplify it as a mathematical model:

$$
n=m^{2}, m=1,2,3 \ldots \text { or } m \in N
$$

The entries of a SuDoku is from a set of $m^{2}$ distinct symbols where are arranged in such way that every symbol occurs exactly once in each column, each row and each $m$ $\times m$ region. These rules are similar to Latin Squares rules but there exists one extra rule which states that the symbols in an $m \times m$ region also must occur not more than once.

Latin Square and SuDoku both are $n \times n$ array. But the size of SuDoku is specified as $n=m^{2}$ which $m$ is a positive integer. Therefore, we can conclude that SuDoku is subset of Latin Square. With the additional rule of SuDoku, it can be said that SuDoku is a special case of Latin Square (Weisstein, 2006). Any solution to a SuDoku puzzle is a Latin Square (Wikipedia, 2006). Figure 1.3 shows an example of $9 \times 9$ SuDoku.


Figure 1.3 An example of $9 \times 9$ SuDoku

From an order $n$ Latin Square, if the rows for that Latin Square are rearranged, a different Latin Square will exist. It will give the same result if the columns are rearranged. So, if the procedure is repeated with $n$-1 times, $n$-1 new Latin Square will be obtained. Consequently, these Latin Squares will form a cube (call it Latin Cube) if
they are combined together. This is the main idea of the method that will be discussed in this research. Figure 1.4 shows a Latin Cube.


Figure 1.4 A Latin Cube with $n=4$

The same idea will be applied in SuDoku. But, SuDoku consist an extra rule which leads to a higher complexity to find the matching layers. The interchanging of rows and columns will be applied at the same time to a SuDoku. The purpose is to eliminate the possibility of clashing among the elements in an $m \times m$ region. Figure 1.5 shows an example of $4 \times 4 \times 4$ SuDoku Cube.


Figure 1.5 An example of SuDoku Cube with $m=2$ and $n=4$

From Figure 1.3 and Figure 1.4, the Latin square and SuDoku seem just lying on the surface of cube. In fact, it does not only occur on the surface of the cube. If the cube is separated into layer, every layer, no matter vertical or horizontal, a Latin Square or SuDoku will be obtained respectively. It means that the cube is formed by combining $n$ layers of Latin Square or SuDoku where $n \in N$. It is hard to imagine or view the Latin Cube and SuDoku Cube in 2D form. The Latin Cube and SuDoku Cube will be viewed in a 3D environment, in Chapter 4.

### 1.2 OBJECTIVES OF RESEARCH

This research has four main objectives:
i. To determine whether different layers of Latin Square can form a Latin Cube that fulfils Latin Square rules.
ii. To determine whether different layers of SuDoku can form a SuDoku cube that fulfils SuDoku rules.
iii. To create a program that can produce a Latin Cube and SuDoku Cube respectively, by obtaining the first layer.
iv. To investigate whether a Latin Cube or SuDoku Cube is unique or not.

### 1.3 SCOPE OF RESEARCH

This research just covers the method of finding matching layers to form a Latin Cube or SuDoku Cube. This research will not touch on methods that are used to solve a Latin Square or a SuDoku. The methods that are discussed in this research also can not be used to solve any Latin Cube or SuDoku Cube. It is only about how to generate
and visualize the Latin Cube and SuDoku Cube. The size of Latin Cube can up to $80 \times$ $80 \times 80$. The number larger than that is not allowed because of the limitation of compiler, processor and system memory. The size of SuDoku Cube can up to $64 \times 64$ $\times 64(m=8)$. The size $81 \times 81 \times 81(m=9)$ SuDoku Cube is not allowed because of the same reason.

### 1.4 HYPOTHESIS

The methods discussed in this research will find the matching layers for a Latin Square or SuDoku to form a Latin Cube and SuDoku Cube. The set of matching layers will not be unique because different Latin Cube and SuDoku Cube can be formed from an initial layer.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 LATIN SQUARE

In 1783, a great mathematician, Leonhard Euler introduced Latin Square as a "nouveau espece de carres magiques", means a new kind of a familiar puzzle, Magic Square (Bogomolny, 2006). Euler used Latin as the name of Latin Square because he used Latin characters as the symbols in a square (Wikipedia, 2006).

A reduced Latin Square or sometimes called normalized or in standard form if the elements in first column and first row are in natural order (Wikipedia, 2006). Figure 2.1 shows an example of reduced Latin Square and Figure 2.2 shows an example that is not a reduced Latin Square.

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 2 | 1 | 4 | 3 |
| 3 | 4 | 1 | 2 |
| 4 | 3 | 2 | 1 |

Figure 2.1 An example of reduced Latin Square

| 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- |
| 4 | 1 | 2 | 3 |
| 3 | 4 | 1 | 2 |
| 2 | 3 | 4 | 1 |

Figure 2.2 An example of not a reduced Latin Square

The Latin Square in Figure 2.1 is said to be reduced because the first row and first column also $\{1,2,3,4\}$. The example in Figure 2.2 is not a reduced form because the first column is not in natural order. A not reduced Latin Square can be changed to reduced form by permuting or reordering (Wikipedia, 2006). This permuting or reordering concept is applied in this research to obtain different Latin Squares from an initial Latin Square.

When a game, especially a puzzle game becomes famous, the mathematician will try to calculate all the number of possible puzzle. For Latin Square, all the possible Latin Squares can be computed by using the following formula:

$$
[(n!)(n-1)!] \times \mathrm{R}
$$

where $n$ is the size of a Latin Square and R is the number of reduced Latin Square for certain size of Latin Square (Wikipedia, 2006). Table 2.1 shows the number of reduced Latin Square for different size.

The total number of possible Latin Square is needed to know so that the possible Latin Cube for a certain order can be calculated. For example, the possible Latin Cube for an order 4 Latin Square should not be more than 576 because a dimension is added. This is important when find a Latin Cube in certain order. The
total number can be estimated before the cube can be found. Table 2.2 shows the total number of Latin Square for $n=1 \ldots \ldots .11$.

Table 2.1 The number of reduced Latin Square with different $n$

| $n$ | R |
| :---: | ---: |
| 1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |
| 5 | 56 |
| 6 | 9408 |
| 7 | 16942080 |
| 8 | 377597570964258816 |
| 9 | 7580721483160132811489280 |
| 10 | 5363937773277371298119673540771840 |
| 11 |  |

Table 2.2 Total number of Latin Square

| $n$ | Total number of Latin Square |
| :---: | ---: |
| 1 |  |
| 2 | 1 |
| 3 | 2 |
| 4 | 12 |
| 5 | 576 |
| 6 | 161280 |
| 7 | 8982437658213039871725064756920320000 |
| 8 | 55124751496156892842531225600 |
| 9 |  |
| 10 |  |
| 11 |  |

### 2.2 SUDOKU

This section will discuss about a brief history of SuDoku, some basic strategies to solve the SuDoku, total number of possible SuDoku grid and Dion Cube, which is where the idea of SuDoku Cube came from.

### 2.2.1 Brief History of SuDoku

SuDoku is a puzzle that is similar to Latin Square. This puzzle was designed by Howard Garns, a retired architect and freelance puzzle constructor (Maxwell, 2005). The main idea came from the Latin Square that created by Euler, but Howard Garns added an extra rule for it, which the entries in every $m \times m$ region must occur no more than once (Carter, 2006).

SuDoku puzzle was first published in 1979 at U.S. (Wikipedia, 2006). The SuDoku Puzzle was first published by the specialist puzzle publisher in New York, Dell Magazines, in its magazine Dell Pencil Puzzles and Word Games. They put the title as "Number Place" (Wikipedia, 2006). So, in U.S., SuDoku puzzle is also known as "Number Place" (Santos-Garc'1a \& Palomino, 2006).

In April 1984, the SuDoku puzzle was first introduced in Japan by Nikoli, in the paper Monthly Nikolist and known as "Suuji wa dokushin ni kagiru" (Timmerman, 2006). SuDoku can be translated as "the numbers must be single" or "the numbers must occur only once" (Winkler, 2005). The puzzle was named by Kaji Maki, the
president of Nikoli. At a later date, the name was abbreviated to SuDoku where Su means number and Doku means single or odd (Wikipedia, 2006).

### 2.2.2 Strategies to Solve SuDoku

Usually, there are a few strategies to solve a SuDoku puzzle. The first strategy is scanning. Scanning consist two basic techniques, which are cross-hatching and counting. Cross-hatching, being the scanning of rows or columns, to identify which line in a particular region may contain a certain number (Lexico Publishing Group, 2006). Then, the process is repeated among the rows and the columns. Counting is used to identify a missing number in a row, column or an $m \times m$ region.

The second strategy is marking-up. This method is used when all the blank cells have more than one candidate. Then, a blank cell can be marked up with the possible candidate. In this situation, the last strategy can be used, which is analyzing. The what-if approach is using here. This means that a blank cell can be chosen and then make a guess to select a candidate number (Lexico Publishing Group, 2006). If the number that had chosen is the solution, then, the solving progress can be continued. Otherwise, these two strategies will be repeated.

SuDoku requires no math ability. It is a puzzle of logic and will not involve the mathematics operation such as adding and subtracting. So, players do not need to master in mathematics when they play the puzzle (Kingfisher Books, 2005).

### 2.2.3 Total Number of Possible SuDoku Grid

Similar to the Latin Square, many people are interested in finding the total number of possible SuDoku grid. Felgenhauer and Jarvis (2005) wrote a paper that enumerates the total number of possible SuDoku grids. In this paper, they calculated that SuDoku have 6670903752021072936960 possible grids.

This calculation is based on the standard $9 \times 9$ SuDoku. The formula to get this number showed as below:

$$
\begin{aligned}
9!\times 72^{2} \times 2^{7} \times 27,704,267,971 & =6670903752021072936960 \\
& \approx 6.671 \times 10^{21}
\end{aligned}
$$

The last factor is a prime number (Wikipedia, 2006). This formula was derived by using logic theory. The total number of possible SuDoku grid is observed because the purpose is same with the purpose to know the total number of Latin Square.

### 2.2.4 Dion Cube

In May 2005, Dion Church, a Telegraph SuDoku enthusiast (Wikipedia, 2006) comes up with an "ultimate SuDoku", which is a three dimension SuDoku. This 3D SuDoku is called the Dion cube. The entire rules that apply in 2D SuDoku also apply in 3D SuDoku. The Dion cube is first published in the Daily Telegraph in May 2005. (Taalman, 2006). Figure 2.3 shows an example of Dion Cube.

## REFERENCES

Bogomolny, A. 2006. Latin Square. Retrieved 30 May 2006 from http://www.cut-theknot.org/arithmetic/latin.shtml.

Carter, E. 2006. The SuDoku Master - Advanced Strategies \& Visual Techniques. Lulu Press, Inc.

Crownover, E. \& Thibault, J. 2005. Latin Square. Retrieved 30 May 2006 from http://web.usna.navy.mil/~wdj/teach/sm463/thibault-crownover-latinsquares.pdf.

Felgenhauer, B. \& Jarvis, F. 2005. Enumerating possible Sudoku grids. Retrieved 31 May 2006 from http://www.afjarvis.staff.shef.ac.uk/sudoku/sudoku.pdf.

Kingfisher Books. 2005. SuDoku. Kingfisher Publications Plc.

Lexico Publishing Group, LLC. 2006. Sudoku wikipedia encyclopedia reference. Retrieved 30 May 2006 from http://www.reference.com/browse/wiki/Sudoku.

Maxwell, K. 2005. Macmillan English Dictionary Word of The Week Archive Sudoku. Retrieved 31 May 2006 from http://www.macmillandictionary.com/ New-Words/051107-Sudoku.htm.

Santos-Garc'ia, G. \& Palomino, M. 2006. Solving Sudoku Puzzles with Rewriting Rules. Electronic Notes in Theoretical Computer Science. Retrieved 30 May 2006 from www.elsevier.nl/locate/entcs.

Taalman, L. 2006. POW \#11: 4x4x4 Sudokube. Retrieved 31 May 2006 from http://www.math.jmu.edu/~taal/POW/POW11.html.

Timmerman, C. 2006. THE EVERYTHING 30-MINUTE SUDOKU BOOK. F+W Publications, Inc., United State.

Weisstein, E. 2006. Latin Square. Wolfram Mathworld. Retrieved 31 May 2006 from http://mathworld.wolfram.com/LatinSquare.html.

Wikipedia. 2006. Latin Square. Wikimedia Foundation, Inc. Retrieved 1 June 2006 from http://en.wikipedia.org/wiki/Latin_square.

Wikipedia. 2006. Sudoku - Wikipedia, The Free Encyclopedia. Retrieved 1 June 2006 from http://en.wikipedia.org/wiki/Sudoku.

Winkler, M. 2005. Super Sudoku 360 Volume 1. Brain Kandy Books, United State.

