

VISUAL PROJECTION OF PLANES, VECTORS, SURFACE AREAS AND
APPLICATION OF INTEGRALS USING 3D VRML

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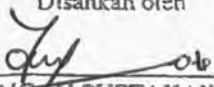
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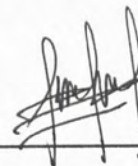
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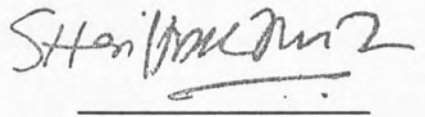
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ABSTRACT

The interpretations of mathematical calculations are often based on traditional methods of using pens and papers to visualize graphical views. These methods are further enhanced by using a visualization program to help students to achieve better understanding through visual interpretations. Virtual Reality Modeling Language (VRML) is the tool that is used in this project to assist in the study of objects' characteristics, as well as provide a platform that resembles the Cartesian coordinates. Transformations such as translation, rotation and scaling are implemented in this project to provide different viewpoint of objects and their behaviour under such changes. The fundamentals of this project are based on the manipulation of different objects that shares certain similarities and uses such changes to better explain these similarities. The emergence of technologies such as immersive virtual reality (IVR) provides students with the ability to interact with graphic objects defined in the virtual world. Implementation of this project revolves around line and vectors in space, surfaces areas and application of integrals. All of these depend heavily on graphical view to explain how their equations behave in 3D space.



PENGIMEJAN VISUAL PERMUKAAN RATA, VEKTOR, LUAS PERMUKAAN DAN APLIKASI PENGKAMIRAN MENGGUNAKAN VRML 3D

ABSTRAK

Intepretasi terhadap fungsi-fungsi serta persamaan geometri adalah berdasarkan kepada model tradisional, iaitu penggunaan alat-alat seperti pen and kertas untuk menghasilkan rajah-rajah untuk pemahaman secara visual. Kaedah ini berpotensi untuk diperbaiki dengan menggunakan alat perisian komputer untuk menghasilkan gambarajah secara tiga dimensi (3D). Virtual Reality Modeling Language (VRML) merupakan perisian komputer yang digunakan dalam projek ini untuk membantu dalam pemahaman ciri-ciri objek yang ditetapkan oleh pengguna. Selain itu, perisian komputer ini menyediakan suatu dasar yang bercirikan koordinat Cartesian. Transformasi seperti translasi, putaran serta penskalaan untuk mengkaji objek-objek dari sudut penglihatan yang berbeza. Konsep-konsep yang digunakan di dalam projek ini adalah berdasarkan kepada keupayaan pengguna untuk memanipulasikan objek-objek yang mempunyai persamaan dari segi bentuk objek dan persamaan matematik objek-objek tersebut. Perkembangan teknologi dari segi reality maya membolehkan pengguna berinteraksi dengan objek yang ditakrifkan. Implementasi di dalam projek ini berlingkar dalam skop seperti vektor di dalam tiga dimensi, permukaan objek serta aplikasi pengkamiran. Kesemua skop ini memerlukan pemahaman visual untuk mengkaji ciri-ciri objek di dalam ruang tiga dimensi.



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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Virtual Reality Modeling Language (VRML) is a descriptive language which allows the creation of non-immersive virtual environments. VRML is an open platform-independent file format which encodes computer-generated graphics in such way that it is easily transported across the Internet network. To view the 3D graphics, the user will be required to have a special web browser which is able to display graphics and stimulate 3D “environments” or “worlds”. Beside that, it also enables users to move and interact with objects and also visualize from different angles. Part of the capabilities of VRML is the ability to link documents objects or other 3D worlds that could be viewed in the browser itself.

VRML is designed to create 3D worlds on the World Wide Web which enable users to manipulate and view from all possible angles (Ranga *et al.*, 1999). The text format used in VRML is based on the description of ASCII text description. A previous research has signified the effectiveness of teaching difficult concepts to learners (Taxen *et*



al., 2002). VRML is a function-based modeling, which is able to unify different types of mathematical functions, designed to enrich web visualization techniques for modeling. Parametric, implicit and explicit functions are utilized to define geometry, color, texture, and other properties of shapes. Other than that, it also allows user to perform geometry transformations such as scaling, rotation, and translation on the 3D objects, apart from the capabilities to enhance the visualization with the addition of colors, textures, sounds, backgrounds, lights and animation (Liu *et al.*, 2006).

Since the 3D space defined in VRML consists of 3 dimensional coordinates and locations, which is similar to any geometry representation using the Cartesian Coordinates x, y, z axes, it is a suitable program to incorporate theories found in mathematical problems. VRML acts as a platform for users to represent the objects desired at the corresponding coordinates without any alteration from what they have learned in theory. They can be applied directly into the computer code procedures and the corresponding objects will be projected at the desired location. Unlike other programs such as C++ Builder, the users will need to convert their objects coordinates into certain values before the objects could be displayed at the desired location in its visualization (Wikipedia, 2006b).

The purpose of this dissertation is to integrate the concepts of mathematical theories found in planes, vectors, conic sections and applications of integral, which will require a virtual visualization method to interpret the beauty of these mathematical formulas across learners. Since VRML has the capabilities to define geometry shapes in



its 3D browser, it would be an advantage especially in the education field to utilize this program in teaching mathematics.

1.2 REAL LIFE VS FREE CHOICE

The advantage of using VRML is that it provides unlimited 3D program for the existing systems of PC, which has been a common usage in every house and classes. The definition of the 3D space is basically infinite, although technically is it only limited by the size of the monitor screen. VRML can be used to support school education and provide a constructive and progressive learning theory (Liu *et al.*, 2006).

Most of the learning experiences are enhanced by the involvement of the students. Such traditional methods of teaching by using only books are quite passive and reduce the interactions between students in their participation in the learning process. On the other hand, by utilizing the capabilities of VRML, students can actually interact and manipulate objects which are projected from the program (Liu *et al.*, 2006).

As such, the aid of using computer graphics will increase the choices of better learning opportunities. This program not only serves as a tool, but also a platform for students to visualize objects or matters which are difficult to describe using traditional tools such as papers and pens. This becomes obvious when it comes to objects, which is hardly understood without the aid of such program (Liu *et al.*, 2006).



1.3 INTERACTIVE VS NON-INTERACTIVE

As compared to the traditional methods of teaching, which is still common among schools, students are having difficulties to understand the concepts and ideas which are being presented by the instructors or lecturers. These matters will thus limit their understandings throughout the entire courses. The limitation of traditional methods in teaching is that the concept is static and non-interactive. This further highlights the importance of using electronic media to present concepts in a more interactive way (Liu *et al.*, 2006).

There is a significant importance of using electronic media for construction and communications of ideas in technical fields. Over the past years, the science of mathematics has advanced as well as the computational techniques. Fields such as numeric computations, geometry coordinates and application of integrals are evolving from 2D representation to 3D visualizations. Furthermore, the identification of new functions having widespread importance in emerging applications requires a platform to effectively present ideas and rules behind these mathematical importance (Wang *et al.*, 1999).

1.4 SYSTEMATIC CODE PROCEDURES

As compared to other visualization programs such as C++, VRML is a suitable platform to analysis 3D objects which are visualize on the computer screen and has a degree of



freedom to conduct the design analysis over these projections. Researchers could utilize VRML capabilities to render 3D objects through the Internet as a collaboration medium for different users to view and study the models. VRML has a prefix order of which definitions should be used on after another (Ranga *et al.*, 1999). Figure 1.1 shows an example of the VRML source code.

```

Transform {
  Translation  -4 -0.25 0
  children [
    Anchor {
      url  ["line intersect.wrl"]
      children [
        Shape {
          appearance Appearance {
            material Material {
              diffuseColor 1 0 0
            }
          }
          geometry Sphere  { radius 0.5 }
        }
      ]
    }
  ]
}

```

Figure 1.1 Example of VRML coding for defining a sphere in the 3D space.

From the mentioned examples, keywords such as *Transform*, *translation*, *children*, *anchor*, *url*, *shape*, *appearance*, *material* and *geometry* are defined in such order that it could not be executed if the previous link is missing. To clarify this statement, we assume that “*Transform*” is missing from the entire code procedure, the VRMLPad will detect the error and thus the whole code procedure could not be executed.

The mentioned line will be highlighted and users will be prompted to correct the code procedure accordingly.

Cybermath is crash prone and difficult to configure. However, VRML has the advantages to minimize compiler errors when it reads the code procedures before executions. The function of geometry is to create objects with other properties such as size, radius and height. Other examples of such definitions are cylinders, circles, cones and pyramids, lines and polygons faces (Taxen *et al.*, 2002).

VRML is easy to learn as such that it creates a 3D space which utilized the x, y, z axes. The statement is shown through the usage of “translation” from Figure 1.1, followed by three variables, each representing the axis respectively. By applying mathematical concepts into VRML, one could easily define the 3D space which is often mentally challenging to interpret and comprehend the courses.

1.5 VISUALIZATION METHODS

Data analysis depend mostly on the visualization mediums to aid users to discover and analyze patterns using simple and multi-dimension data sets, simplified and pictorial forms of representations are often preferred over the complex of endless streams of numbers, tables and symbolic strings. This matter has become increasing problematic if the courses require the representation of 3D or multi dimension graphics as it is



complicated to illustrate over the traditional method of hand drawing using pens and papers as its mediums (Sangole *et al.*, 2003).

Since it is much easier to understand data values by using visual aid, VRML is a suitable program where it is capable of transforming raw two-dimensional (2D) plots into a higher degree of 3D representations that could reveal the underlying patterns of the original data sets. Often, this kind of visualization involves certain of doubts of inaccuracy of the data being presented. VRML particularly is to assist students to comprehend the relationships between data sets.

1.6 OBJECTIVE OF STUDY

In this dissertation, the objectives are to:

- i. Visualize mathematical functions into geometric representations.
- ii. Aid in teaching through visualization of lines, planes, surface areas and applications of integrals.
- iii. Assist in the study of the properties of the function-based geometry.

1.7 SCOPE OF STUDY

In this dissertation, lines, vectors, planes, surface areas and application of integrals are considered. Planes include two dimensions (2D) and three dimension (3D) which will be used to analysis the properties that lies behind the mathematical functions. The movement



and projections of any given sets of vectors were investigated such as the possibilities of line-crossing in different directions and positions. This is especially important as it is difficult to determine whether two set of lines will cross, which is opposite to planes that in any given planes in a 3D space, the planes will eventually cross each other. For surface areas such as parabolic, hyperbolic, eclipse, circle, sphere, pyramid and cylinder were considered. Some of the surface areas are predefined in the VRML geometry databases such as circles, sphere, cones and cylinders. Other functions such as parabolic and hyperbolic will use a different approach to visualize its graphical representations in the VRML 3D space. This approach will include the definition of value by value of every points along the functions to plot a comprehensible graphs.



CHAPTER 2

LITERATURE REVIEW

2.1 VIRTUAL REALITY MODELING LANGUAGE

There are a number of studies that showed that using electronic media to present ideas and concept of mathematical problems. This is advancement from the traditional way of teaching, as parallel to the tremendous progress in technologies in graphical structures and its easy-to-use applications. Such programs are easily accessible and its application is using simple English language as its medium to define its variables and writing its codes. Such programs are CyberMath, C++ and also VRML.

As function-based geometry increased in dimensions and sizes to solve more advanced level of problems, the upgrade in computer design is coherent to be able to visualize such high degree of concepts. The advantages of these electronic media are that they serve as platforms to provide construction and communication of ideas in technical fields. The rapid growing of identification of new mathematical functions is having widespread importance in emerging applications like constructions, graphic design and architecture. There are also the possibilities to display mathematical concepts using



graphics methods rather than two dimensional (2D) like the traditional methods did (Wang *et al.*, 1999)

The transformation from $R^3 \times R^3$ on different mathematical entities such as points, lines, planes and sphere is applicable using various graphical programs. In their research, they have utilized the capabilities of CyberMath together with the Distributed Interactive Virtual Environment (DIVE). The DIVE is an internet-based multi-user VR system where participants navigate in 3D space and see, meet and interact with other users and applications (Taxen *et al.*, 2002).

Objects that are defined in the virtual 3D environment are easy to be manipulated (translation or rotation) by users to provide better understandings easy to comprehend difficult concepts in mathematics. The arbitrary transformation from $R^3 - R^3$ can be specified and the effects of this transformation can be studied interactively. The usage of graphical concepts in Cybermath has the potential to contribute to a more positive attitude towards mathematics, especially young generations.

CyberMath provides a useful platform to help lecturers to better present their mathematical concepts, but due to the problems of instability and complications, it is not widely preferred by users internationally (Taxen *et al.*, 2002).

The Internet is a suitable platform to display ideas and concepts, which is easily accessible by anyone using computers. It has a wide impact on human lives, business,



entertainment, shopping and communications. Although there is a wide system of sharing information, but mathematics as one of the science branch has been taken for granted of its importance in the development of human knowledge.

VRML is designed as one of the Internet compatible software and is readily available as free open-source programs, which is capable of visualizing objects and provide the users to conduct design analysis of the models. A previous study has utilized the Internet as a collaboration medium to allow multiple users to log on to study VRML models. A visual representation of mathematical design models enhanced the understandings of students especially majoring in mathematics. Furthermore, web based methodology of teaching is used to supplement the classroom teaching for its ubiquitous, inexpensive, economical, and efficient nature to use Internet as a medium of information (Ranga *et al.*, 1999).

The capabilities of VRML to reuse points which share common objects, edges, faces, vertices, colors, and textures reduced the reluctance of crash and increase its usability. VRML, which is text based is also able to link with other programming language such as Practical Extraction and Reporting Language (PERL) to control objects by changing its parameters, thus providing a more dynamic environment. The flexibility will enable users to easily manipulate and define the desired objects, and study them efficiently. They can even share their information using the Internet and get feedback to allow modifications and improvements.



PERL has the ability to link with VRML to modify its parameters and project new objects according to the new variations (Ranga *et al.*, 1999). The design generation is based on the input of data, which will be computed accordingly and a new VRML file will be created for the model. This program also acts as a platform to provide a user interface between the users and VRML files. This will reduce the cumbersome of users to directly read and modify the parameters. PERL is a powerful reading arbitrary text files, are efficient at extracting information and does not require any compilers, which increase its efficiency as a suitable platform.

There is an emphasizing on the usage of various mathematical techniques to transform multivariate data sets into simple graphical objects (Sangole *et al.*, 2003). This is to enhance the ideas of underlying system behavior. Simplified and pictorial forms are preferred representations of complex data over endless streams of numbers, tables, and symbolic behavior. In these ways, it is easier to identify patterns by using visual cues, shapes, and color.

There is a shifting phase from using two dimensional (2D) representations to a more complex and higher level of three dimensions (3D), which is to capture the underlying patterns hidden in the original data sets. This approach is to enhance users' understandings by providing a graphical and pictorial view of the geometries being studied, which enables users to manipulate and view the objects from different angles.



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