SOLVING LINEAR 1-D AND 2-D HEAT EQUATIONS USING ADM

LOW TUCK HWA

A DISSERTATION SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH HONOURS

MATHEMATICS WITH COMPUTER GRAPHIC PROGRAMME
SCHOOL OF SCIENCE AND TECHNOLOGY
UNIVERSITI MALAYSIA SABAH

APRIL, 2007

BORANG PENGESAHAN STATUS TESIS@

JUDUL: Solving Linear 1-D and 2-D Heat Equations using ADM.

Ijazah: Sarjana Muda Sains Dengan Kepujian Matematik Dengan Komputer Grafik

SESI PENGAJIAN: 04/05

Saya LOW TUCK HWA

(HURUF BESAR)

mengaku membenarkan tesis (LPS/Sarjana/Doktor Falsafah)* ini disimpan di Perpustakaan Universiti Malaysia Sabah dengan syarat-syarat kegunaan seperti berikut:

1. Tesis adalah hakmilik Universiti Malaysia Sabah.
2. Perpustakaan Universiti Malaysia Sabah dibenarkan membuat salinan untuk tujuan pengajian sahaja.
3. Perpustakaan dibenarkan membuat salinan tesis ini sebagai bahan pertukaran antara institusi pengajian tinggi.
4. **Sila tandakan (/)

[ ]	SULIT	(Mengandungi maklumat yang berdarjah keselamatan atau kepentingan Malaysia seperti yang termaktub di dalam AKTA RAHSIA RASMI 1972)
[/]	TERHAD	(Mengandungi maklumat TERHAD yang telah ditentukan oleh organisasi/badan di mana penyelidikan dijalankan)
[ ]	TIDAK TERHAD	

Disahkan oleh

Low

(TANDATANGAN PENULIS)

(TANDATANGAN PUSTAKAWAN)

Alamat Tetap: 33-04-05, Flat Sri Johor, 56100, Cheras, Kuala Lumpur

Cik Suzelawati Zenian

Nama Penyelia

Tarikh: 20/4/2007

Tarikh: 20/4/2007

CATATAN: * Potong yang tidak berkenaan.

** Jika tesis ini SULIT atau TERHAD, sila lampirkan surat daripada pihak berkuasa/organisasi berkenaan dengan menyatakan sekali sebab dan tempoh tesis ini perlu dikelaskan sebagai SULIT dan TERHAD.

@ Tesis dimaksudkan sebagai tesis bagi Ijazah Doktor Falsafah dan Sarjana secara penyelidikan, atau disertasi bagi pengajian secara kerja kursus dan penyelidikan, atau Laporan Projek Sarjana Muda (LPSM).

DECLARATION

I declare that this dissertation is the result of my own research, except for the materials referred to as cited in the reference section. The dissertation had not been accepted for any degree and is concurrently submitted in candidature of any degree.

27 April 2007

LOW TUCK HWA

HS2004-4658

CERTIFICATION

Signature

1. **SUPERVISOR**

   (Ms. Suzelawati Zenian) ________________

2. **EXAMINER**

   (Assoc. Prof. Dr. Jumat Sulaiman) ________________

3. **DEAN**

   (Supt./Ks. Assoc. Prof. Dr. Shariff A.K. Omang) ________________

ACKNOWLEDGEMENTS

First of all, I would like to thank my supervisor, Miss Suzelawati Zenian for her kind and helpful supervision in addition to her valuable revisions made to this thesis. Her concerns, guidance and encouragement offered to me throughout this project have eased my life.

Special thanks to all of the other lecturers for their smiles, support, knowledge, direction, understanding and helps in many ways.

All of the coursemates truly deserve a warm thank for their continued moral supports, and encouragement. Thanks are extended to my friends in hometown who had kept encouraging me and made me laugh whenever I had felt frustrated and bored.

Heartiest appreciation to my parents, brother and sister for their never-ending loves and supports throughout my study.

My thesis would never have been finished without these people who continued provides essential knowledge, information, supports, and encouragement. My deepest thanks to all of them.

ABSTRACT

In this study, an efficient numerical method had been investigated. This method is Adomian decomposition method (ADM) in which it was involved in solving 1-D and 2-D linear heat equations. This method has been used to solve the heat equation, which governs on numerous scientific and engineering experimentations. Some cases of heat equations were solved as examples to illustrate the ability and reliability of this method. Approximate analytical solutions calculated by ADM were compared with the exact solutions in order to obtain the absolute errors. The values of absolute error obtained were small and it was proved that ADM had a fast convergence in solving 1-D and 2-D linear heat equation problems. In this case of study, ADM had been proved in providing a facile calculation in the numerical problems solving in heat equations. It can be seen from the demonstration on the proposed method that was successfully produced a better approximation towards the exact solution.

MENYELESAIKAN PERSAMAAN HABA 1-D DAN 2-D MENGGUNAKAN KPA

ABSTRAK

Dalam kajian ini, satu kaedah berangka yang efektif telah dikaji. Kaedah ini ialah Kaedah Penguraian Adomian (KPA) yang mana ia telah digunakan dalam penyelesaian masalah yang melibatkan persamaan haba linear 1-dimensi dan 2-dimensi. Kaedah ini telah digunakan untuk menyelesaikan persamaan haba yang mana ia melibatkan pelbagai masalah yang timbul di kalangan eksperimentasi saintifik dan kejuruteraan. Beberapa kes dalam persamaan haba ini telah diselesaikan untuk menunjukkan kebolehan dan kebolehpercayaan kaedah berangka ini. Penyelesaian analitik penghampiran yang dihitung dengan menggunakan KPA telah dibanding dengan penyelesaian tepat untuk mendapatkan ralat mutlak. Nilai ralat mutlak yang didapati adalah amat kecil dan ini telah membuktikan bahawa KPA mempunyai penumpuan yang pantas dalam menyelesaikan masalah-masalah persamaan haba linear 1-dimensi dan 2-dimensi. Dalam kajian ini, KPA telah dibuktikan mampu memudahkan langkah-langkah pengiraan dalam penyelesaian masalah persamaan haba. Ini dapat ditunjukkan melalui penggunaan kaedah tersebut yang mana ia telah menunjukkan nilai penghampiran yang lebih tepat.

CONTENTS

		Page
DECLARATION		ii
CERTIFICATION		iii
ACKNOWLEDGEMENTS		iv
ABSTRACT		v
ABSTRAK		vi
LIST OF CONTENTS		vii
LIST OF TABLES		ix
LIST OF SYMBOLS		x
CHAPTER 1	**INTRODUCTION**	
1.1	Introduction	1
1.2	Numerical Analysis	1
1.3	Ordinary Differential Equation	2
1.4	Partial Differential Equation	3
1.5	Adomian Decomposition Method	4
1.6	Objective of Study	5
1.7	Scope of Study	6
CHAPTER 2	**LITERATURE REVIEW**	
2.1	Standard Adomian Decomposition Method	7
	2.1.1 Partial Solution	11
	2.1.2 Adomian Polynomials	12
	2.1.3 Noise	12
2.2	Modified Adomian Decomposition Method	13
2.3	Crank-Nicolson Finite Difference Method	14
2.4	Sinc-Galerkin Method	15
CHAPTER 3	**METHODOLOGY**	
3.1	Introduction	17

3.2 1-D Linear Heat Equation 17

3.3 2-D Linear Heat Equation 20

CHAPTER 4 RESULTS AND DISCUSSION

4.1 Introduction 22

4.2 Analysis of Problem 22

4.2.1 1-D Heat Problem 23

4.2.2 2-D Heat Problem 30

4.3 Discussion of Problems 38

4.3.1 1-D Heat Problem 38

4.3.2 2-D Heat Problem 40

CHAPTER 5 CONCLUSION

5.1 Conclusion of Study 41

5.2 Future Study 42

REFERENCES 43

APPENDIX

LIST OF TABLES

No. of Tables		Page
4.1	Numerical results for $\left|u(x,t)-\phi_{30}(x,t)\right|$ where $u(x,t)=2e^{-(\pi^2/4)t}\sin(2\pi x)$.	27
4.2	Numerical results for $\left|u(x,t)-\phi_{20}(x,t)\right|$ where $u(x,t)=2e^{-(\pi^2/4)t}\sin(2\pi x)$.	28
4.3	Numerical results for $\left|u(x,t)-\phi_{10}(x,t)\right|$ where $u(x,t)=2e^{-(\pi^2/4)t}\sin(2\pi x)$.	29
4.4	Comparison of absolute error for $\phi_{10}(x,t)$, $\phi_{20}(x,t)$ and $\phi_{30}(x,t)$.	30
4.5	Numerical results for $\left|u(x,y,t)-\phi_{30}(x,y,t)\right|$ where $u(x,y,t)=e^{-(\pi^2/8)t}\sin(\pi x)\sin(\pi y)$.	34
4.6	Numerical results for $\left|u(x,y,t)-\phi_{20}(x,y,t)\right|$ where $u(x,y,t)=e^{-(\pi^2/8)t}\sin(\pi x)\sin(\pi y)$.	35
4.7	Numerical results for $\left|u(x,y,t)-\phi_{10}(x,y,t)\right|$ where $u(x,y,t)=e^{-(\pi^2/8)t}\sin(\pi x)\sin(\pi y)$.	36
4.8	Comparison of absolute error for $\phi_{10}(x,y,t)$, $\phi_{20}(x,y,t)$ and $\phi_{30}(x,y,t)$.	37

LIST OF SYMBOLS

$=$	Equal sign
$\approx$	Approximate equal sign
$\{ \}$	Curly bracket
( )	Parentheses
[ ]	Square bracket
$\leq$	Inequality sign; less than or equal to
$\geq$	Inequality sign; greater than or equal to
$\langle$	Inequality sign; less than
$\rangle$	Inequality sign; greater than
.	Multiplication sign
$+$	Addition sign
-	Subtraction sign
/	Division sign
$\sum$	Summation sign
$\int$	Integral sign

CHAPTER 1

INTRODUCTION

1.1 Overview

Differential and integral equations, known as the theory that describes the world are not only appear in the physical science, but in biology, sociology, and all the scientific disciplines that attempt to make us know more about the world in which we live. But many of the equations that govern the physical world have no solutions in closed form. Therefore, in order to find the answer to questions about the world we live on, we need resort to solving these equations numerically.

1.2 Numerical Analysis

Numerical analysis is called the mathematics of scientific computing. This involves the study, development, and analysis of algorithms for obtaining numerical solutions to various mathematical problems. In many problems this implies producing a sequence of approximations; thus the questions involve the rate of convergence, the accuracy (or even validity) of the answer, and the completeness of the response. Numerical solutions to differential equations require the determination not of a few numbers but of an entire function; in particular, convergence must be judged by some global criterion.

1.3 Ordinary Differential Equation

An ordinary differential equation is an equality involving a function and its derivatives. An ODE of order n is an equation of the form shown by equation 1.1 as below:

$$F(x, y, y', \ldots, y^{(n)}) = 0, \tag{1.1}$$

where y is a function of x, $y' = dy/dx$ is the first derivative with respect to x, and $y^{(n)} = \frac{d^n y}{dx^n}$ is the nth derivative with respect to x.

An ODE of order n is said to be linear if it is of the form

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \ldots + a_1(x)y' + a_0(x)y = Q(x) \tag{1.2}$$

A linear ODE where $Q(x) = 0$ is said to be homogeneous. Confusingly, an ODE of the form shown by equation 1.2 as below

$$y' = f(\frac{y}{x}) \tag{1.3}$$

is also sometimes called "homogeneous."

In general, an nth-order ODE has n linearly independent solutions. Furthermore, any linear combination of linearly independent functions solutions is also a solution. Simple theories exist for first-order (integrating factor) and second-order (Sturm-Liouville theory) ordinary differential equations, and arbitrary ODEs with linear constant coefficients can be solved when they are of certain factorable forms. Integral transforms such as the Laplace transform can also be used to solve classes of linear ODEs.

While there are many general techniques for analytically solving classes of ODEs, the only practical solution technique for complicated equations is to use numerical methods. The most popular of these is the Runge-Kutta method, but many others have been developed, including the collocation method and Galerkin method. A vast amount of research and huge numbers of publications have been devoted to the numerical solution of differential equations, both ordinary and partial (PDEs) as a result of their importance in fields as diverse as physics, engineering, economics, and electronics.

1.4 Partial Differential Equation

Partial Differential Equation or so called PDE forms the basic for many problems in science, that to limit the choice of examples. We can know that most of the fundamental laws of physical science are written in terms of partial differential equations. These are present in computer modeling from the hydrodynamic calculations needed for the airplane design, weather forecasting, and the flow of fluids in the human body to the dynamical interactions of the elements that make up a model economy. A Partial Differential Equation (PDE) is an equation involving functions and their partial derivatives. Although a PDE can be expressed in multiple dimensions, but the smallest number for the illustration is two, which one of them may be time. Many of these examples which describe the aspects of the physical world do have the form shown as equation 1.3 as below:

$$a(x,y)\frac{\partial^2 z(x,y)}{\partial x^2} + 2b(x,y)\frac{\partial^2 z(x,y)}{\partial x \partial y} + c(x,y)\frac{\partial^2 z(x,y)}{\partial y^2} = F\left[x, y, z\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right] \quad (1.4)$$

and such can be categorized into three distinct groups, which are shown as equation 1.5, 1.6 and 1.7 as below:

$$b^2(x,y) - a(x,y)c(x,y) < 0 \quad , \quad \text{Elliptic} \quad (1.5)$$

$$b^2(x,y) - a(x,y)c(x,y) = 0 \quad , \quad \text{Parabolic} \quad (1.6)$$

$$b^2(x,y) - a(x,y)c(x,y) > 0 \quad , \quad \text{Hyperbolic} \quad (1.7)$$

1.5 Adomian Decomposition Method

Adomian Decomposition Method or so called ADM, is a numerical method that introduced by George Adomian since 1980s. ADM is a creative and effective method for exactly solving functional equations of various kinds. It is important to note that a large amount of research work has been devoted to the application of the ADM to a wide class of linear and nonlinear, ordinary or partial differential equation (Zhang *et. al.*, 2006). The efficiency and effectiveness of the ADM for many kinds of equations are well known (Kaya & El-Sayed, 2004).

ADM is the most powerful tool for calculation of the analytic solutions of the linear or nonlinear PDE, a method introduced by Adomian (Bildik & Bayramoglu, 2005). ADM also been discovered to obtain solution of linear / nonlinear fractional diffusion and wave equation. Analytical methods commonly used to solve wave equation and diffusion equation, are very restricted and numerical techniques involving discretization of the variables on the hand gives rise to rounding off errors. ADM provides a solution in terms of a rapidly convergent power series, has been proven successful in deriving analytical solution of linear and nonlinear differential equations. This method is preferable over numerical methods as it is free from rounding off errors and neither requires large computer power and memory (Jafari & Daftardar, 2006).

1.6 Objectives of The Study

The main objectives of this study are

(a) To present an efficient numerical method called Adomian decomposition method that facilitates the calculation in numerical problems solving.

(b) To examine the accuracy of the presented numerical method against exact solution.

(c) To investigate whether higher iteration used able to give higher accuracy to the approximate analytical solution.

1.7 Scope of Study

For this case of study, it will be focused on solving heat equation. This heat equation will be concerned on the type of linear and in two different stage which are 1-dimensional (1-D) and 2-dimensional (2-D). The numerical method used in solving these problems is standard Adomian decomposition method (ADM). The accuracy of standard ADM will be examined by comparing the approximate analytic solutions with the exact solutions. The approximate analytic solutions will be also compared between 10 iterations, 20 iterations and 30 iterations.

CHAPTER 2

LITERATURE REVIEW

2.1 Standard Adomian Decomposition Method

The particular exact solutions of a linear heat equation and a nonlinear heat equation that usually arises in mathematical biology are obtained in both x and t directions using Adomian decomposition method reported in Pamuk (2005). In their research, they found that the approximate solutions obtained by using Adomian Decomposition Method (ADM) are very close to the partial exact solutions of linear and nonlinear heat equations. By using only 20 terms of the decomposition series, they achieve a very good approximation to the partial exact solution. This show that the speed of convergence of ADM is very fast and overall errors can be made pretty small by adding new terms to the series.

Bildik & Bayramoglu (2005) used ADM to investigate nonlinear two dimensional wave equation. The analytic of the nonlinear wave equation is calculated in the form of a series with easily computable components. The nonhomogenous equation is effectively solved by employing the phenomena of the self-canceling "noise" terms where the sum of the components vanishes in the limit. Besides that, they also compare ADM with some known techniques and shows that the present approach is powerful and reliable. By comparing the errors occur in all the comparison methods, they stated that the decomposition scheme obtains efficient results much closer to the actual solutions. It is also worth noting that the advantage of the decomposition methodology shows a fast convergence of the solution.

Phenomena in electromagnetic, acoustics, viscoelasticity, electrochemistry and material science are also described by differential equations of fractional order. The solution of the differential equation containing fractional derivatives is much involved. In the study of obtaining the solution of a nonlinear fractional differential equation, Saha & Bera (2005) applied the use of ADM. After getting the solutions, they compared the obtained solution with those obtained by truncated series method. As a result, they found that the advantage of the decomposition method is that it does not change the problem into a convenient one for the use of liner theory. Moreover, no linearization or perturbation is required in using ADM. Besides that, the decomposition method is straight forward, which without restrictive assumptions and the components of the series solution can be easily computed using any mathematical symbolic package. Therefore, ADM provides more realistic series solutions that generally converge very rapidly in real physical problems.

In order to solve porous media equation that usually occurs in nonlinear problems of heat and mass transfer, and in biological systems, Pamuk (2005) used ADM. We can find these problems such in description of unsteady heat transfer in a quiescent medium with the heat diffusivity being a power-law function of temperature. The discussion stated out ADM obtains a very accurate numerical solution for the nonlinear problems in comparison with other methods such as Lie symmetry reduction method and antireduction method. The numerical results they have obtained justify the advantage of this method by only needed use a few term approximation. It also does not required large computer memory and discretization of the variables t and x.

The systems of nonlinear algebraic equations more often are not solved analytically hence the resort to numerical solutions. Indeed, there is no general theory for finding their solutions. Kaya and El-Sayed (2004) using ADM to develop an approximation of the solution of the systems of nonlinear algebraic equation $F(x) = 0$. Special cases of the system of nonlinear equations are solved using the algorithm of the decomposition method. It turns out that the convergence of this algorithm is rapid. ADM has the advantages of being more concise for numerical purpose; since it provides the solution as an infinite series in which each term can be easily determined, and the series is quickly convergent towards an accurate solution.

Nonlinear phenomena that appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics can be modeled by partial differential equations. A broad class of analytical

In order to solve porous media equation that usually occurs in nonlinear problems of heat and mass transfer, and in biological systems, Pamuk (2005) used ADM. We can find these problems such in description of unsteady heat transfer in a quiescent medium with the heat diffusivity being a power-law function of temperature. The discussion stated out ADM obtains a very accurate numerical solution for the nonlinear problems in comparison with other methods such as Lie symmetry reduction method and antireduction method. The numerical results they have obtained justify the advantage of this method by only needed use a few term approximation. It also does not required large computer memory and discretization of the variables t and x.

The systems of nonlinear algebraic equations more often are not solved analytically hence the resort to numerical solutions. Indeed, there is no general theory for finding their solutions. Kaya and El-Sayed (2004) using ADM to develop an approximation of the solution of the systems of nonlinear algebraic equation $F(x) = 0$. Special cases of the system of nonlinear equations are solved using the algorithm of the decomposition method. It turns out that the convergence of this algorithm is rapid. ADM has the advantages of being more concise for numerical purpose; since it provides the solution as an infinite series in which each term can be easily determined, and the series is quickly convergent towards an accurate solution.

Nonlinear phenomena that appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, mathematical biology and chemical kinetics can be modeled by partial differential equations. A broad class of analytical solutions methods and numerical solutions methods were used in handle these problems. Then a discussion on a new application of ADM for nonlinear physical equations has been made by El-Wakil and Abdou (2006). They investigated the behavior of Adomian solutions and the effects of different values of t. The ADM has been proved to be effective and reliable for handling differential equations, linear or nonlinear. Besides that, ADM has been used to solve effectively, easily, and accurately a large class of linear and nonlinear equations with approximates which converge rapidly. It is unlike classical techniques that needed the nonlinear equations to be transforming before they can be solved.

The problems arise in integral equations has been discussed in Babolian and Davari (2004) and also Biazar *et al.* (2003). They propose some new ideas to implement ADM to solve integral equations and consider linear and nonlinear systems of integral equations of the first kind respectively. According to Biazar *et al.* (2003), they used ADM to solve the linear and nonlinear systems of Volterra integral equations of the first kind.

Numerical methods, which are commonly used as characteristics method, needed a large size of computation works and usually the round-off error causes the loss of accuracy. According to Biazar and Ebrahimi (2005), they applied ADM to solve hyperbolic partial differential equations and make up a results comparison with the characteristics method. The results of shows that ADM can provide a more approximate solution by a smaller size of computation compare to characteristics method. Since ADM has been applied to solve many functional equations and systems of functional equations, we can see that ADM is such a very effective method and results in considerable saving in computation time. Besides this, ADM is very sensitive to initial conditions.

For the Burger's-Huxley and Burger's-Fisher equation solving, Ismail *et al.* (2004) obtained approximate solutions by using ADM. They study these problems based on initial value problem. The obtained results presented only few terms of the expansion are required to obtain the approximate solution which is found to be accurate and efficient. Nevertheless ADM is a powerful method which provided an efficient potential for the solution of the physical applications modeled by nonlinear differential equations. The algorithm can be used without any need to complex calculations except for simple and elementary operations. Hashim *et al.* (2006) applying ADM into solving Lorenz system. A comparison between the decomposition solutions with the fourth-order Runge-Kutta (RK4) is made. ADM yields, without linearization, perturbation, transformation or discretization, an analytical solution in terms of a rapidly convergent infinite power series with easily computable terms.

2.1.1 Partial Solution

This partial solution technique is found by Adomian and Rach (Abbasbandy, 2007) in solving PDE problems. They noticed that considerable work is saved by solving only one of the linear operator equations. The partial solution technique used in ADM is applied in Luo *et al.* (2006). This application is considered in solving heat and wave equations. They solved these equations with partial solution technique, in cases where boundary conditions are "well-defined", the solution can be obtained by the usual decomposition method. But in cases where the boundary conditions are "ill-defined", approximate expansion of the initial term should be used.

2.1.2 Adomian Polynomials

The Adomian polynomials are the technique build by Adomian by using special kinds of polynomials. The Adomian polynomials were given without mathematical formulas, which make difficult to deploy especially when high orders are required. Then in the study by Abdelwahid (2003), they introduced a mathematical formulas for Adomian polynomials. By using Mathematica packages, they have shown that these formulas can be used to generate any desired orders of Adomian polynomials. Then another research also had done on Adomian polynomial which conducted by Chen and Lu (2004). They presented a reliable technique for calculating Adomian polynomials for nonlinear operators. In order to mechanized the Adomian decomposition method in solving differential equations, the algorithm made fulfilled with symbolic computation.

2.1.3 Noise

Related phenomena were recently established to facilitate the convergence of the solution or to make savings in the computational work. Then, Adomian and Rach introduced phenomena of the so-called noise terms. These noise terms were defined as the identical terms with opposites signs that appear in the first two components of the series solution of

$u(x)$. It was concluded that if a term or terms in the component u_0 are canceled by a term or terms in the component u_1, even though u_1 contains further terms, then the remaining non-canceled terms in u_0 may provide the exact solution $u(x)$. The noise terms appear always for inhomogeneous equations. Then for more recently, a necessary condition that is essentially needed to ensure the appearance of the noise terms in the inhomogeneous equations was developed by Wazwaz and El-Sayed (2001).

The necessary condition for the noise terms to appear in the components u_0 and u_1 is that the exact solution $u(x)$ must appear as part of u_0 among other terms. In addition, the remaining non-canceled terms of u_0 must satisfy the equation under discussion. The new necessary condition developed by Wazwaz was formally proved and tested to demonstrate the power of the noise terms whence these terms appear in the components u_0 and u_1 (Wazwaz & El-Sayed, 2001)

2.2 Modified Adomian Decomposition Method

In the study conducted by Babolian and Biazar (2002), they modify the standard Adomian decomposition method for solving of the nonlinear equation. A comparison was made between the solution obtained by using standard ADM and those by using modified Adomian methods. Then they found that the modified method can be studied for functional equations, and also can be extended to a system of nonlinear equations, after

REFERENCES

Abbasbandy, S. 2007. A numerical solution of Blasius equation by Adomian's decomposition method and comparison with homotopy perturbation method. *Chaos, Solitons and Fractals* **31**, 257-260.

Abdelwahid, F. 2003. A mathematical model of Adomian polynomials. *Applied Mathematics and Computation* **141**, 447-453.

Al-Khaled, K., Kaya, D. & Aslam Noor, M. , 2004. Numerical comparison of methods for solving parabolic equations. *Applied Mathematics and Computation* **157**, 735-743.

Babolian, E. & Biazar, J. 2002. Solution of nonlinear equations by modified adomian decomposition method. *Applied Mathematics and Computation* **132**, 167-172.

Babolian, E. & Davari, A. 2004. Numerical implementation of Adomian decomposition method. *Applied Mathematics and Computation* **153**, 301-305.

Biazar, J. & Ebrahimi, H. 2005. An approximation to the solution of hyperbolic equations by Adomian decomposition method and comparison with characteristics method. *Applied Mathematics and Computation* **163**, 633-638.

Biazar, J., Babolian, E. & Islam, R. 2003. Solution of a system of Volterra integral equations of the first kind by Adomian method. *Applied Mathematics and Computation* **139**, 249-258.

Bildik, N. & Bayramoglu, H. 2005. The solution of two dimensional nonlinear differential equation by the Adomian decomposition method. *Applied Mathematics and Computation* **163**, 519-524.

Bradie, B. 2006. A Friendly Introduction to Numerical Analysis with C and MATLAB Materials on Website. Pearson Education, Inc., New Jersey. 803-882.

Chen, W. H. & Lu, Z. Y. 2004. An algorithm for Adomian decomposition method. *Applied Mathematics and Computation* **159**, 221-235.

David Kincaid & Ward Cheney. 2002. Numerical Analysis: What is it? Numerical Analysis: Mathematics of Scientific Computing, 3rd Edition, 1-2.

Dehgham, M. 2001. Crank-Nicolson finite difference method for two-dimensional diffusion with an integral condition. *Applied Mathematics and Computation* **124,** 17-27.

El-Wakil, S. A. & Abdou, M. A. 2006. New applications of Adomian decomposition method. *Chaos, Solitons and Fractals*.

Hashim, I. *et al.* 2006. Accuracy of the Adomian decomposition method applied to Lorenz system. *Chaos, Solitons and Fractals* **28**, 1149-1158.

Hosseini, M. M. 2006. Adomian decomposition method with Chebyshev polynomials. *Applied Mathematics and Computation* **175**, 1685-1693.

Ismail, H. N. A., Raslan, K. & Rabboh, A. A. A. 2004. Adomian decomposition method for Burger's –Huxley and Burger's-Fisher equations. *Applied Mathematics and Computation* **159**, 291-301.

Jafari, H. & Daftardar-Gejji, V. 2006. Solving linear and nonlinear fractional diffusion and wave equation by Adomian decomposition. *Applied Mathematics and Computation*.

Kadalbajoo, M. K. & Awasthi, A. 2006. A numerical method based on Crank-Nicolson scheme for Burgers'equation. *Applied Mathematics and Computation.*

Kaya, D. & El-Sayed, S. M. 2004. Adomian decomposition method applied to systems of nonlinear algebraic equations. *Applied Mathematics and Computation* **154**, 487-493.

Luo, X. G., Wu, Q. B. & Zhang, B. Q. 2006. Revisit on partial solutions in the Adomian decomposition method: Solving heat and wave equations. *J. Math. Anal. Appl.* **321**, 353-363.

Pamuk, S. 2005. An application for linear and nonlinear heat equations by Adomian's decomposition method. *Applied Mathematics and Computation* **163**, 89-96.

Pamuk, S. 2005. Solution of the porous media equation by Adomian's decomposition method. *Physics Letters A* **344**, 184-188.

Saha Ray, S. & Bera, R. K. 2005. An approximate solution of a nonlinear fractional differential equation by Adomian decomposition method. *Applied Mathematics and Computation* **167**, 561-571.

Wazwaz, A. M. & El-Sayed, S. 2001. A new modification of the Adomian decomposition method for linear and nonlinear operators. *Applied Mathematics and Computation* **122**, 393-405.

Zhang, B. Q., Wu, Q. B. & Luo, X. G. 2006. Experimentation with two step Adomian decomposition method to solve evolution models. *Applied Mathematics and Computation* **175**, 1495-1502.