FAIR MASTERMIND

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DECLARATION

I hereby declare that this dissertation contains my original research work. Sources of findings reviewed herein have been duly acknowledged.

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ABSTRACT

This dissertation introduces a modification of the two-player code-breaking game Mastermind with the introduction of cheating by the Code-maker and asking by the Code-breaker. The element of cheating and asking provide a fair setting where the Code-maker is able to cheat with the markers awarded to the Code-breaker whilst the Code-breaker is able to ask the Code-maker to determine the occurrence of a cheat. These actions are done once respectively during the first three guesses. Game trees are constructed and tables are drawn to see the possible combinations available when the actions of cheating and asking take place. The best time for the Code-maker to cheat would be the 2^{nd} guess, when his opponent is at a juncture of confusion. However, though there are many options in markers to use for cheating, the best strategy is to use only logical markers to avoid doubt in his opponent. The criterias for the cheating markers to be convincing is that the markers must go in line with his opponent's 1^{st} guess and gives possible combination of codes for his opponent's attempt at the 3^{rd} guess. The best times for the Code-breaker to ask would be at the 1^{st} guess due to the lack of markers to suspect as well as the 3^{rd} guess due to more information on the set. However, the Code-breaker must use logical thinking and intuition in his suspicions on what he believes to be the rightful markers to his guess in order to break the code.

MASTERMIND SAKSAMA

ABSTRAK

Disertasi ini merupakan modifikasi ke atas permainan dua-pemain membongkar kod Mastermind dengan pengenalan konsep menipu oleh Pencipta-kod dan konsep menyoal oleh Pembongkar-kod. Elemen penipuan dan penyoalan membolehkan peluang saksama antara pemain di mana Pencipta-kod dapat menipu dengan penandaan yang diberikan kepada Pembongkar-kod manakala Pembongkar-kod dapat menyoal unutk menentukan kejadian penipuan. Penipuan dan penyoalan dibenarkan hanya sekali masing-masing dalam tempoh tiga cubaan yang pertama. Gambar rajah pokok dan jadual digunakan untuk memaparkan kombinasi kod yang ada selepas penipuan dan penyoalan berlaku. Masa menipu yang terbaik untuk Pencipta-kod adalah ketika cubaan kedua di mana lawannya berada dalam kemusykilan. Namun demikian, walaupun terdapat pelbagai cara untuk menipu dengan penandaan, strategi terbaik adalah dengan menggunakan penandaan yang logikal untuk mengurangkan kemusykilan pihak lawan. Kriteria untuk meyakinkan pihak lawan adalah dengan memberi penandaan yang selari dengan penandaan cubaan pertama di samping memberi kemungkinan bagi kombinasi kod untuk cubaan ketiga. Masa menyoal yang terbaik untuk Pembongkar-kod adalah ketika cubaan pertama disebabkan kurangnya cara penandaan untuk disyaki serta cubaan ketiga di mana terdapat lebih pengetahuan dalam set permainan. Namun demikian, Pembongkar-kod harus menggunakan pemikiran logikal yang disertai dengan intuisi dalam penekaan penandaan yang disyaki adalah penandaan yang sewajarnya dalam usaha membongkar kod.

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LIST OF ABBREVIATIONS

C	Cheating
SM	Suspected Markers
SM1	Suspected markers for 1st guess
SM2	Suspected markers for 2nd guess
SM3	Suspected markers for 3rd guess
NB	Not believe the markers
NB1	Not believing the markers at the 1st guess
NB2	Not believing the markers at the 2nd guess
NB3	Not believing the markers at the 3rd guess
A	Ask
B	Believe the markers
B1	Believe markers at 1st guess
B2	Believe markers at 2nd guess
B3	Believe markers at 3rd guess
XXX	No possible codes

CHAPTER 1

INTRODUCTION

1.1 Preface

Mastermind is a simple yet fascinating two-player code-breaking board game invented in 1970 by an Israeli postmaster and telecommunications expert, Mordecai Meirowitz (Wolfram, 2006). Initially his idea of this code-breaking game of deduction was turned down by many of the leading toy companies, but adamant and refusing to give up, he took it to the International Toy Fair at Nuremberg in February 1971. There he showed the game to a small English company, Invicta Plastics Ltd and the Leicester based company bought up the entire intellectual property rights to the game and under the guidance of its founder, Mr. Edward Jones-Fenleigh, refined it and released it in 1971-72.

1.2 The Contents of Mastermind

The Mastermind unit consists of 96 Code pegs in 8 colours, 15 small black or red Indicator pegs and 15 small white Indicator pegs.

The game is played using:

i) decoding board, with a shield at one end covering a row of four large holes, and twelve additional rows containing four large holes next to a set of four small holes.

ii) code pegs of six different colours, with round heads, which will be places in the large holes on the board.

iii) key pegs, some coloured, some white, which are flat-headed and smaller than the code pegs; they will be places in the small holes on the board.

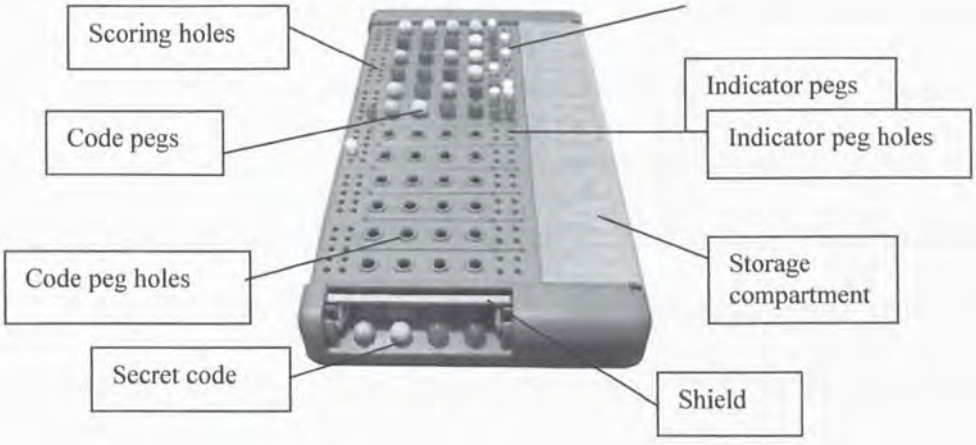

Figure 1.1 Mastermind game (Wikipedia, 2006).

1.3 Rules and Regulations in Original Mastermind

Mastermind is a game where the Code-maker and Code-breaker endeavour to outwit each other in a setting of rules agreed upon by both parties. The Code-maker will lay out a secret combination of colored pegs (digits or other symbols are allowed), which is hidden from the sight of the Code-breaker. The game kicks off with the Code-maker

selecting a code, a sequence of four colours (c_1, c_2, c_3, c_4) chosen from a set of six colours where two or more pegs of the same colour is allowed (repetition allowed).

Once the code is set, the Code-breaker or also known as the Decoder, will draw pegs from a set and will make a guess (g_1, g_2, g_3, g_4) aimed at duplicating the exact colours and positions of the hidden combination. After every guess, the Code-maker must inform the Code-breaker of his progress as follows; with a black marker (for exact match) for each peg that is in the correct position and right color or with a white marker (for near match) for each peg with the correct colour but in different position. To summarize the interplay of exact matches and near matches in mathematical language, consider first, the number of exact matches which is the number of times $c_k = g_k$, $k = 1,..., 4$. Let n_i be the number of times colour i is in the code and m_i is the number of times i is in the guess. Then colour i is matched min (n_i, m_i) times (Temporel, 2003). The total number of colour matches is the sum of matches for all individual colours. Since exact matches are evaluated first, the number of near matched is computed by;

$$\sum_{i=1}^{6} \min(ni,mi) - (\text{the number of exact matches})$$

Evaluation of exact matches has precedence over evaluation of near matches (Nelson, 2000). The formula used to get the amount of possible codes is

Possible codes = colours $^{\text{no of holes}}$

Suppose the pegs are red(R), blue(B) ,cyan(C) , yellow(Y) , green (G), purple (P) , orange (O) and the indicator markers white (W) and black (B).

Code maker's secret sequence	●	●	●	●
Player's first guess	●	●	●	●

In this example, the Code-maker would indicate that the player has one peg of the right colour in the right position and one peg of the right color in the wrong position but would give no clue that the cyan peg (C) and the purple peg (P) are the relevant choices. So the response markers would be 1 black and 1 white. If a guess contains the same colour in more than 1 position, but the secret code contains it only once, it is an implicit rule that is based on the assumption that each response marker refers to one and only one coloured peg (Greenwell, 2006).

1.4 Problem Statement

Mastermind has seen many modifications ever since it has been marketed globally. There are even online versions of the game created to replace the human versus human board game of Mastermind. In the advanced version of Mastermind, the Code-maker is allowed to leave one or more blank spaces when setting the code. This means that instead of the 8 colours in the Mastermind unit, the Code-breaker has an equivalent of 9 colours from which to choose (Hasbro, 2000).

Cheating in online computer games is a broad category of activities, all of which are generally regarded as modifying the game experience in a way that gives a player an unfair advantage over the other players. Different activities constitute cheating, as it is often a matter of consensus opinion and it usually depends on the game in question (Wikipedia, 2006). Fair Mastermind hopes to deal with cheating albeit in a different manner. Since allowing the Code-maker to cheat would be to the

Code-maker's advantage of withholding the code, it is a fair play when his opponent, the Code-breaker gets a chance to ask the Code-maker. Mastermind is interesting because it provides avenues in which one can play around with notions of quality of different questioning strategies. These strategies could be generalized to more general question-answer settings (Kooi, 2003). The strategy that minimizes the expected number of guesses required, thus maximizing the Code-breaker's expected payoff is the best winning strategy to undertake (Kooi, 2005).

1.5 Objectives of Study

There are a few main objectives that propelled interest in this study. It is hoped that the objectives could be realized with the completion of this study.

i) Creating a fair play

With the main idea of placing both opponents on equal footing, hence the name Fair Mastermind, this study allows the Code-maker to cheat and the Code-breaker to ask the Code-maker in any one of his first 3 guesses once, respectively.

ii) Minimizing the Code-breaker's doubts

Cheating creates an environment of uncertainty and doubt. Therefore introduction of the concept of asking will minimize the Code-breaker's doubts.

iii) Finding optimal ways of cheating and asking

It is hoped that this study will seek the best ways for the Code-maker to cheat and the Code-breaker to ask.

iv) An Interactive Game

This study involves verbal actions [asking (Code-breaker) and answering (Code-maker)] and this dynamic setting makes the game interactive.

1.6 Scope of Study

This modification is chosen based on a few parameters that play important roles in creating a challenging and interesting game.

i) This study considers 4 colours [Red(R), Green (G), Blue (B) and Cyan(C)] and 3 holes.

ii) Maximum 8 guesses to break the code. Cheating and asking (once respectively) are allowed only in the first 3 guesses of the game.

iii) The number of type of codes will remain the same in the original Mastermind game, but the maximum guesses will increase. If in the original Mastermind game, for 4 colours and 3 holes game, the possible codes are $4^3 = 64$. But in this game, the maximum guesses will be

more than the original although the possible guesses available are the same. Since there are 64 possible combinations when guessing the secret code and 8 possible guesses are allowed, it is moderately unlikely that the Code-breaker can guess the code with relative ease.

iv) In this study, the secret code chosen is RGC and the first guess would be RRR.

v) The Code-breaker is given a time limit of 8 minutes to break the code, failing which he loses the set of game being played. A chess-clock will be used where the distribution of time is 3 minutes for the Code-maker and 5 minutes for Code-breaker.

vi) In the analysis of cheating and asking for the 2^{nd} guess, the codes used are the possible codes when the 1^{st} guess is not a cheat and is believed by the Code-breaker. Meanwhile for the 3^{rd} guess, the codes used are the possible remaining codes when the 1^{st} and 2^{nd} guess is not a cheat and those guesses are believed by the Code-breaker.

1.7 Preface of Fair Mastermind

In Fair Mastermind a total of 8 guesses are allowed where the Code-maker is only allowed to cheat once in the first three guesses of the game. Meanwhile the Code-breaker is allowed to ask only once during that period of cheating

i) The Code-maker is allowed to choose any one of the first three guesses of the Code-breaker to cheat on. During that period, the Code-maker can cheat only once.

ii) The Code-maker can only cheat on answers than the real answer (RGC=secret code).

iii) Fair Mastermind is still under the rules if the Code-breaker manages to find the secret code before the Code-maker has a chance even to cheat. This scenario may occur when the Code-maker decides to cheat beforehand at the 2nd or 3rd guess of the Code-breaker but the Code-breaker by strategy or plain luck stumbles on the secret code at the 1^{st} or 2^{nd} guess.

iv) Cheating on an answer does not signify that the whole answer is wrong in all cases. If the Code-breaker's 1st guess in breaking the secret code (RGC) is RRR and the Code-maker decides to cheat at his 1st guess by not giving any markers, it is not an indication that the whole answer is wrong. It could contain either 1R or 2Rs. However it cannot be 3Rs because the Code-maker cannot cheat on any guesses by means of treating it as a secret code.

v) In the original Mastermind, there is an element of luck in picking random choices that might help in revealing the secret code. However if a player decides to depend on luck in this game, this element of luck

would be needed in his timing in asking his Code-maker as well as his suspected markers.

viii) In the original Mastermind, the Code-maker can only consider 1R if the real code contains only 1R. This implicit rule has to be adhered to in instances other than cheating.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Mastermind is a two-player game which utilizes the element of strategies in an endeavour to break the hidden code within the most limited guesses (Roger, 1999).

2.2 Representation of a Game

Mathematically, there are a myriad of ways of categorizing and describing games. There are formal elements in common for games and these elements are adhered to in game settings.

i) a list of players

ii) a complete description of what the players can do (their possible actions)- the players alternate in their moves and are subjected to rules about what moves can be made in any given configuration of the board game.

iii) descriptions of what the players know when they act- players observe each other's moves and hence keep track of the game's progress.

iv) a specification of how the players' actions lead to outcomes

v) a specification of the players' preferences over outcomes (Watson, 2004).

2.2.1 The Uncertainty of Games: In the Context of Mastermind

A crucial aspect of the specification of a game involves the information that players have when they choose strategies. A source of uncertainty arises when players do not have the same information about the current state of the game, so that one player may not possess all the information that is available to the totality of players. The uncertainty about the further course of a game can be based on some factors. In comprehending these uncertainties in game theory, the three main types of mechanism at work are chance, the large number of combinations of different moves and the different states of information among the individual players (Bewersdorff, 2005).

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