

MATHEMATICS OF CHESS RATING

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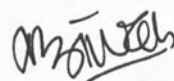
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DECLARATION

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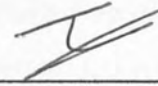


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ABSTRACT

Chess is an abstract strategy board game designed for two player and chess is a game with rating which indicates the playing strength of a player. Chess rating consist of numbers which represent a person's playing ability. There are numerous rating system which are Ingo system, Edo rating system, Elo system, Sonas system and Glicko rating system. Rating is one of the aspects that make the game of chess interesting and because of this ratings need to be as accurate as possible. The main objective of this study is to compare and to choose a more accurate alternative rating system, between Sonas rating system and Glicko rating system, besides the existing Elo rating system that are used by chess federations. The existing rating for the selected players, which are Elo rating will be set as the standard rating. Sonas rating and Glicko rating will therefore be compared against Elo rating. Data for this research were obtained from World Chess Federation (FIDE) official website, which consists of information on the selected world's top five professional chess players. Three calculations on the formulas were done on the five top players, which are Elo rating formula using new Elo table as proposed by Alliot J. M., Sonas rating formula and Glicko rating formula. The results obtained were summarized into graphs using Microsoft Office Excel 2003 and the graphs shows that Glicko rating formula produces ratings that are closest to Elo rating. Ratings calculated from Sonas rating formula were inconsistent and the difference between Sonas rating and Elo rating is very large. In contrast, the difference between Glicko rating and Elo rating are small. Therefore, it is concluded that Glicko rating formula is the preferred rating system, other than Elo rating formula.

ABSTRAK

Permainan catur adalah satu permainan yang berasaskan strategi and konsentrasi yang dicipta untuk dua orang pemain. Catur adalah permainan dengan pemeringkatan yang menggambarkan tahap kebolehan seseorang itu bermain catur. Pemeringkatan catur hanyalah terdiri daripada nombor-nombor yang mewakili kebolehan seseorang pemain itu dalam permainan catur. Terdapat beberapa kaedah sistem pemeringkatan yang telah wujud sekarang iaitu sistem Ingo, sistem pemeringkatan Edo, sistem Elo, sistem Sonas dan sistem pemeringkatan Glicko. Pemeringkatan adalah salah satu aspek yang menjadikan permainan catur amat menyeronokkan dan oleh sebab itu, pemeringkatan perlulah setepat yang boleh. Objektif utama kajian disertasi ini adalah membanding and memilih satu sistem pemeringkatan alternatif selain daripada sistem pemeringkatan Elo yang telah digunakan oleh persekutuan catur di seluruh dunia. Sistem pemeringkatan Elo akan dijadikan sebagai piawai di mana sistem Sonas dan sistem Glicko akan dikaji dan dibanding dengan sistem Elo. Data-data bagi kajian ini diperolehi daripada laman web rasmi Persekutuan Catur Sedunia (FIDE) yang terdiri daripada informasi bagi lima pemain catur profesional di peringkat antarabangsa. Pengiraan tiga formula iaitu sistem pemeringkatan Elo menggunakan jadual Elo baru yang diperkenalkan oleh Alliot J. M., sistem pemeringkatan Sonas dan sistem pemeringkatan Glicko akan dibuat ke atas kelima-lima pemain catur yang telah dipilih. Graf akan dilukis menggunakan Microsoft Office Excel 2003 berdasarkan keputusan yang diperolehi daripada pengiraan. Graf menunjukkan bahawa sistem pemeringkatan Glicko menghasilkan pemeringkatan yang paling dekat dengan piawai, manakala sistem pemeringkatan Sonas menghasilkan pemeringkatan yang tidak konsisten. Dengan itu, dapat disimpulkan bahawa sistem pemeringkatan Glicko adalah sistem pemeringkatan yang ideal, selain daripada sistem pemeringkatan Elo.



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CHAPTER 1

INTRODUCTION

1.1 What is Chess Rating?

Chess is a game with rating. A rating system is a system of measuring player's strength and chess playing ability. A player gets rating by participating in chess tournaments. Based on their performance in chess tournament, statistical formulas are used to calculate the ratings for chess player (Spanier, 1984). A player must participate in certain amount of tournament and for a period of years before they are taken into the account of counting their rating. Rating is one of the things that make chess different from any other games (Spanier, 1984). Rating can lead to a more exciting two-player game, but rating is temporary as rating can change, after winning, drawing or losing in tournament based on the player and the opponent's rating. The good news is that once a chess player is rated, he or she will always be rated (FAQ Starting Out, n.d).

Chess rating is useful for pairing up players in tournaments. Players that have the most probability to win in the tournament would not be paired against each other during the earlier round of the competition. Ratings are also helpful for determining

tournament sectioning and prize eligibility, where there is a section in tournaments that may only allow players of specified rating level to compete for section prize.

Other than that, ratings can also be used as a qualifying system for the players to play in privileged tournament and events (Glickman *et al.*, 1999). Rating system is developed to serve many purposes. Ratings make it possible for determining pairing in tournaments to allow similar strength players compete against one another.

Titles are different from rating. Anyone who participated in rated tournaments and beat another player can get a rating but not everyone who participated in rated tournament can get titles. But players with high ratings are almost certainly to have titles like Super Grandmaster, Grandmaster, International Master, and more. Some titles are also awarded to chess solvers, composers and correspondence chess players (Chess Titles, 2006). In order for a player to get a title, he or she must reach a minimum requirement of rating level for different kind of titles (FAQ Starting Out, n.d). Other than reaching certain rating level, one must achieve the required 'norms' in order to get a title (Spanier, 1984).

1.2 Objectives of Research

The objectives of this research are:

- i) To study Elo rating system, Sonas rating system and Glicko rating system.



- ii) To calculate and to study the difference of player's rating using the new Elo table as proposed by Jean-Marc Alliot.
- iii) To compare between Sonas rating system and Glicko rating system against Elo rating system.
- iv) To choose a more accurate alternative rating system, between Sonas rating system and Glicko rating system, other than Elo rating system.

1.3 Scope of Research

This research focuses on three rating system, which are Elo rating system, Sonas rating system and Glicko rating system. For the analysis of the player's rating, information were gathered from World Chess Federation (FIDE) official website. The top five chess players for July 2006, which consist of male players from different countries around the world, were chosen. The chess tournaments that were played were classical games.

Full reports on their performance in tournament from the period of July 2000 until July 2006 were collected. Individual calculations for each tournament were collected from the period of October 2001 until July 2006. FIDE place greater emphasis to more recent games, which results that individual calculations for the past tournaments were not available.

FIDE uses Elo rating system to rate their players. In this research, the players' ratings that were calculated using Elo rating system will be set as the standard rating as Elo rating are widely use by chess federations worldwide. Players' new ratings that

will be calculated using Sonas and Glicko rating system will be compared against Elo rating. The data obtained will be compared and analysed.



CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to the Game of Chess

Chess, as defined by Oxford Advanced Learner's Dictionary is a game for two people played on a board marked with black and white squares on which each playing piece (representing a king, queen, castle, etc.) is moved according to special rules; and the aim is to put the other player's king in a position from which it cannot escape. Chess is a beautiful game and it is inspiring to see a situation where generation gap, society classes and race are invisible because chess can be seen playing in all over the world, by different races, different level of ages, and different classes of the society.

The game of chess involves concentration and powerful thinking. Chess is fun, but it is very hard to master. The game of chess involved many rules and regulation and in order to master all the rules, players must practice and take the risk of losing. By losing, one can learn from their mistakes and eventually become a better player. As a player become stronger, he or she should seek out matches with other stronger player to improve their skills. Chess is no fun if players if there are no risk of losing.



2.1.1 History of Chess

Chess is the oldest strategy game played in the world. Chess was believed to be inherited from the ancient game of chataranga, which was played in western part of India, at around year 450 A.D (Pandolfini, 1992). Chataranga and chess shared some characteristics as chataranga “was played on an eight-by-eight-square board, usually with alternating light and dark squares; it used units of different colors, powers, shapes, and sizes; it empowered units to capture each other, rather than just displace each other; it concluded when a particular unit was captured and/ or rendered immobile; and it did not necessarily require to throw the dice” (Pandolfini, 1992).

2.1.2 Chess Equipment

The equipments involved in a game of chess are chess board, chess pieces, and chess clock. A game of chess is played on a checkered board, and made up of sixty-four individual squares in eight rows of eight squares and the board should be positioned with the corner of each player’s right is light square (Quah *et al.*, 1995), or simpler to say “white on right”. Player with white pieces will make the first move, and then followed by player with black pieces (Quah *et al.*, 1995). The two players will take turn for their move until the time runs out indicate that the end of the game.

The pieces involved in a game of chess are two knights, two bishops, two rooks, one queen, one king and eight pawns, per side. The king is the most important part of chess but yet not the most powerful one (Mack, 2002). The chess pieces and position of every chess pieces is represented in Figure 2.1 shown below,



Figure 2.1 Symbols and positions of chess pieces

(E. I. 01A. Laws of Chess, n.d.)

The chess clock is an essential part of a chess game in competitions and tournaments. Chess clock is invented to keep players from abusing time and to plan his playing time wisely (Quah *et al.*, 1995). A chess clock has a two clock faces and only one knob can be fully press at one time. When the player had made his move, the knob of the chess clock on his side must be pressed and this will release the knob of his opponent (United States Chess Federation, n.d.). The game is consider lose if one of the player run out of time, or there is a checkmate on the board (United States Chess Federation, n.d.).

2.1.3 Objective of the Game

The objective of chess is to win the game and to get a rating, or more, which is to get a title. For chess player to win a game of chess, he or she must check the opponent's king piece. Check is a position where the king is under direct attack. The player must somehow escape his king from the check by either capturing the piece that is attacking



the king, running away, or block the attacking piece with another piece (Pandolfini, 1992). The other position is called checkmate where the king is in direct attack from the opponent's piece. Checkmate is a position where this ends the game without the king being captured by the opponent (Pandolfini, 1992).

Chess can take a lifetime to master but it will be a wonderful moment if a player discovered a surprising move. Anyone can play chess, challenge themselves and improve at any level, and that is the greatest thing of playing chess.

2.1.4 Variants of Chess

Chess variants are forms of chess where the game is played with special pieces, different rules or different board (Wikimedia Foundation Inc., 2006a). A normal type of chess, classical chess that are played in tournament, event and national competition uses two hours for each players, for them to play 40 moves. The players will get additional time of 20 moves every hour if the game has not finished in two hours time (Quah *et al.*, 1995).

Modern chess is played for 90 minutes time control for each player, per game (Sonas, 2002). Blitz chess is a shorter version of chess where each side has three to fifteen minutes for them to move their pieces, and a chess clock is used to limit the time control for each player (Wikimedia Foundation Inc., 2006a). If exceeded the time limit, the game is lost. Bullet chess or 'lightning chess' is a more faster version where the time control is less than three minutes. These types of chess game require players to spend less time thinking (Wikimedia Foundation Inc., 2006a). Rapid chess is chess



game with time control ranges from about 25 minutes to 60 minutes each side (United States Chess Federation, n.d.). Rapid and Blitz game are used as tiebreakers, even in world championship level competition (Sonas, 2002).

There is also Suicide chess, where the goal of the game is to lose all of opponent's pieces and if a piece can be taken, it must be taken by the opposing side. There is also Bughouse chess, where two teams of players play against each other and give the captured pieces to their partner. The other type of chess, Progressive chess is another type of chess game where the number of pieces can increase each turn, for example, white moves one piece, black moves two, white moves three, black moves four and so on. In Nuclear or Atomic chess, other than removing the captured piece from the board after the capture, the capturing piece and every other piece of both players positioned in any adjacent square to that capture were also removed from the board (Wikimedia Foundation Inc., 2006a).

2.2 Introduction to Chess Rating

A rating is an estimate of player's playing strength. As chess player will practice again and again to improve their skill before participating in tournament games, his or her skills, chess understanding, confidence level and concentration will improve. A rating is a mathematical figure that recognize a chess player's playing ability. An improved playing skill and increased confidence level is far more satisfying than gaining rating.

Ratings are statistic, and ratings depend only on the data provided by pairings and outcomes (Jones, 2006). Rating chess player follows the concept of "paired

comparison” where the outcome will indicate a degree of preference of one player over the other (Glickman, 1995). Chess game will produce the result of two players being compared to determine which player is stronger or better. Paired comparison also occur in other sports where two competitors are involved such as football, basketball and hockey. Paired comparison can be described as a function,

$$s : \{(i, j) \in X \times X : i \neq j\} \rightarrow \{0,1\} \quad (2.1)$$

Let i and j be two different items from universe X consisting of N different stimuli. The outcome of the results between i and j can be denoted as s_{ij} , with $s_{ij} = 1$ if $i > j$, where i is preferred over j , and $s_{ij} = 0$ if $i < j$. In chess game, chess players are stimulus-items, and s_{ij} are the outcomes of games played between two players, i and j (Blécourt, 1998).

The first rating statistician was William Byland, and he was originated from Texas. He is known to do all the calculations by hand (Sloan, n.d.). Now, the calculations are done by computer. The formulas were inserted into a program and the computer provide the answer once the information were entered into the computer.

The rating system that chess clubs and chess federations were using is the Harkness system, introduced by Kenneth Harkness (Pandolfini, 1992). This chess rating system had been existed for ten years before Professor Arpad Elo modified and improved the existing system into Elo System (Sloan, n.d.). In the Harkness system, a player will still be able to gain points although the player loses every game in a tournament (Glickman, 1995).

2.2.1 Ingo System

The first rating system was the Ingo System, developed by Anton Hoesslinger in year 1948. He was originated from Federal Republic of Germany (Jones, 2006). The name 'Ingo' was named after the place of origin, Ingolstadt. The basic formula for Ingo System is,

$$R = ER_c - (Pct - 50) \quad (2.2)$$

where,

R is the player's recent rating,

ER_c is the arithmetic average of the opponent's ratings, and

Pct is the player's score, in percentage points.

Ingo System is different from any other rating systems because lower rating means that the player has greater playing strength. As mentioned by Jones (2006), this theory is ridiculous because Anton Hoesslinger relied on his intuition when he developed this rating system. In result, if a player lose in every game in tournaments, he will still be able to gain rating points. Ingo system did not apply any statistical theory in its formula, even though the rating it produces were consistent with individual ranking of chess players (Glickman, 1995).

2.2.2 Edo Rating System

Edo rating system attempted to make the best estimate and the most reliable of the ratings. Edo ratings are calculated using iterative method just like Elo rating system but Edo rating treat each player in every year as a separate player because chess player's ability in playing will change over some time. This make it difficult to apply the "standard static paired comparison experiment" for a long period of time because one's playing abilities changes and there is no direct comparison can be made between players of different period of time. In Edo system, Bradley-Terry Model is used to assure the rating consistency, balance between performance ratings in between the given rating period and neighbouring rating period (Edwards, n.d.). Bradley-Terry model was introduced by statistician Ralph Bradley and M. Terry in 1952 (Glickman, 1995).

Edo rating system is done by obtaining "maximum-likelihood" estimates of ratings between pairs of players that relates scores to difference of ratings. Thus, Bradley-Terry model, a static iterative method, is applied to the all collection of players. This model allows calculation of optimum rating of any set games between players by simple iteration method. Bradley-Terry model is shown as below,

$$s = \frac{n}{1 + 10^{\left(\frac{-(R-r)}{400}\right)}} \quad (2.3)$$



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