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APPLICATION OF CHECK DIGIT SCHEME IN
CHECKING THE VALIDITY OF A
CREDIT CARD NUMBER

CHARMAINE LORAIN TALALLA

MATHEMATICS WITH ECONOMICS PROGRAMME
SCHOOL OF SCIENCE AND TECHNOLOGY
UNIVERSITI MALAYSIA SABAH

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CHARMAINE LORAIN TALALLA

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DECLARATION

I hereby declare that this dissertation contains my original research work. Sources of findings reviewed herein have been duly acknowledged.

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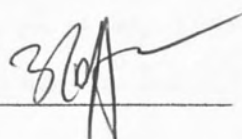
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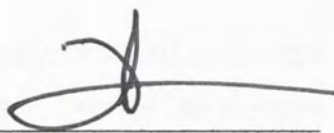


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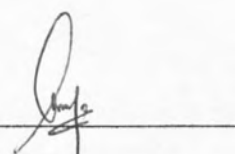
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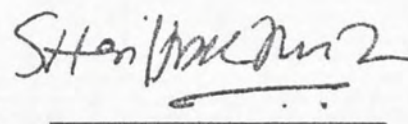
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APLIKASI SKEMA ANGKA KAWALAN UNTUK MENYEMAK KESAHIHAN SUATU NOMBOR KAD KREDIT

ABSTRAK

Kajian ini berfokuskan aplikasi skema angka kawalan, yang dikenali sebagai skema IBM dan digunakan oleh dua jenama kad kredit utama di Malaysia, iaitu MasterCard dan Kad Visa. Angka kawalan ialah digit persepuluh yang ditambahkan kepada suatu nombor kad kredit, yang mewakili digit terakhir nombor kad kredit tersebut. Fungsi angka kawalan ialah untuk menyemak kesahihan suatu nombor kad kredit dan mengesan kesilapan pada suatu nombor kad kredit bila nombor tersebut dihantarkan. Pengiraan angka kawalan dapat dilakukan secara manual dengan menggunakan skema IBM. Pengiraan manual suatu angka kawalan melibatkan beberapa kaedah matematik yang meliputi mod 10, permutasi, putaran notasi, kumpulan simetri, modulo aritmetik dan kekongruenan. Penyemakan kesahihan suatu nombor kad kredit dapat dilakukan secara manual dan digital. Dengan melakukan penyemakan manual, jumlah hasil digit-digit nombor kad kredit mesti menghasilkan suatu nilai yang merupakan ganda 10 yang tepat, maka ini memenuhi persamaan penyemakan dan pada masa yang sama menyatakan bahawa nombor kad kredit tersebut adalah sah. Suatu program C dibangunkan untuk lebih membantu sebagai penyemakan digital dalam menentukan kesahihan suatu nombor kad kredit. Mod 10 merupakan mod yang sesuai digunakan dalam skema IBM. Jika sebarang mod yang lebih kecil dari mod 10 digunakan, ia mengurangkan julat digit persepuluh yang boleh dianggap oleh suatu angka kawalan. Sebaliknya, sebarang mod yang lebih besar dari mod 10 menghasilkan angka kawalan berdigit dua dan meningkatkan julat digit persepuluh yang boleh dianggap oleh suatu angka kawalan. Konsep kebarangkalian diaplikasikan dalam menentukan kebetulan meneka suatu Nombor Pengenalan Peribadi (PIN) berdigit empat berdasarkan bilangan tekaan yang dibuat dan bilangan PIN yang mungkin dikira.



ABSTRACT

This research focuses on the application of the check digit scheme, known as the IBM scheme and used by the two major credit card brands in Malaysia, the MasterCard and Visa Card. A check digit is a decimal digit affixed to a credit card number, which represents the last digit of the credit card number. The function of a check digit is to check the validity of a credit card number and to detect errors in a credit card number when the number has been transmitted. The computation of a check digit can be done manually using the IBM scheme. The manual computation of a check digit involves various mathematical methods, which include mod 10, permutation, cycle notation, symmetric group, modulo arithmetic and congruence. The validity check of a credit card number can be done manually and digitally. By conducting a manual check, the sum of the resulting digits must produce a value which is an exact multiple of 10, thus this satisfies the check equation and at the same time declares that the credit card number is valid. A C program is developed to further assist as a digital check in determining the validity of a credit card number. Mod 10 is an appropriate mod to be used in the IBM scheme. If any other mod lesser than mod 10 is used, it reduces the range of decimal numbers a check digit can assume. On the other hand, any other mod greater than mod 10 produces double-digit check digits and increases the range of decimal numbers a check digit can assume. The concept of probability is applied in determining the chances of guessing a four-digit Personal Identification Number (PIN) based on the number of guesses made and the number of possible PINs.



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LIST OF SYMBOLS

Z^+	positive integer set
σ	sigma
$=$	equals to
$>$	greater than
\leq	less than or equals to
\geq	greater than or equals to
\neq	inequivalent to
\equiv	congruence equivalence
$\%$	percentage
\in	element of
\rightarrow	rightwards arrow
$\{\dots\}$	set
$ \dots $	modulus
(\dots)	cycle



CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF RESEARCH

A credit card is an instrument of payment issued by a wide range of financial, retail and service organizations such as retail outlets, oil companies, travel and entertainment companies, commercial banks and airlines. It enables the cardholder to make purchase of goods and services at merchant outlets. The credit card can be used by the cardholder at any merchant, either locally or internationally, which displays the same credit card brand as that on the cardholder's credit card. A credit card is a convenient mode of payment as cardholders' can make purchases without using cash. The amount of purchase made by the cardholders is billed to them at a later date.

A credit card is a thin wallet-sized strip of cardboard or plastic, measuring "3 $\frac{3}{8}$ inches long by 2 $\frac{1}{8}$ inches wide" in size and weighs about a fifth of an ounce (Evans & Schmalensee, 1999). It contains cardholders' information such as the cardholder's name and a 16-digit credit card number, known as the Primary Account Number (PAN).



All credit cards are of similar shape and size, referred as ID-1. As specified by the ISO 7810 standard 'Identification Cards – Physical Characteristics', the size of a credit card measures [(85.60 x 53.98 mm) or (3.375 x 2.125 in)], with a thickness of 0.03 inches. The ratio of the ID-1 dimensions is approximately the golden section (also known as the golden ratio).

There has been a remarkable development of credit cards throughout the 19th century. These cards started out as embossed cards, and then proceeded to become magnetic stripe cards. The magnetic stripe cards were equipped with security features, which include the hologram, signature panel and the magnetic stripe. However, the magnetic stripe technology had a number of security flaws, where the cards could be cloned by reading, deleting and rewriting the data on the magnetic stripe. As a result, the number of credit card fraud cases increased rapidly. In order to reduce the number of fraudulent cases, chip-based credit cards were introduced. The additional security feature added was the microcontroller chip. The embedded chip features the Europay MasterCard Visa (EMV) standard. Chip-based credit cards featuring the Europay MasterCard Visa (EMV) standard has gained recognition worldwide.

In the Asia Pacific region, Malaysia emerged as the first country to migrate to EMV-compliant chip-based credit cards in May 2005 (The Star, 2005). The EMV-compliant chip cards have stronger security measures. Its electronic data processing system has a high security data encryption technology, which provides authentication and verification that the cardholder is the legitimate owner of the card. This system helps in



authorizing a card transaction. As a result, total losses from credit card fraud in Malaysia have dipped as much as 84% to 0.12% in the first quarter of 2005 compared to 0.74% in the first quarter of 2004 (The Sun, 2005).

In relation to the security features held by a chip card, a check digit scheme, known as the IBM scheme was introduced by Hans Peter Luhn to check the validity of a credit card number. A check digit is the last digit appended to a credit card number. It functions as an error-detection method in detecting errors humans make when typing a credit card number into the computer. The IBM scheme applies the Luhn-10 algorithm. This scheme is used by MasterCard and Visa card organizations or merchants accepting MasterCard and Visa credit cards based on the goods or services purchased by cardholders.

United Kingdom has taken the implementation of chip cards one step ahead with the introduction of a Personal Identification Number (PIN) through the implementation of the Chip and PIN system. This system features the PIN as an extended security feature which has assisted in the reduction of fraudulent cases. The PIN is entered by a cardholder as a means of user identification and to verify a card transaction process. The PIN functions as a barrier code which is not easy to break as it requires a certain number of possibilities in guessing the right PIN.



1.2 MAIN CREDIT CARD BRANDS USED IN MALAYSIA

The two main credit card brands in Malaysia are MasterCard and Visa.

1.2.1 MasterCard

MasterCard is the most widely accepted and recognized global payment card around the world. It is operated by MasterCard International Incorporated, which is a global payment company owned by member banks. The MasterCard logo consists of red and orange balls with the word MasterCard across it, which is located at the lower right-hand corner of the card. The usage of MasterCard has proliferated over the years, comprising more than 23 000 financial institutions worldwide with consumers in 210 countries and territories (MasterCard, 2005). There are over 24 million acceptance locations around the world. MasterCard handles processing of nearly 32 million card transactions on an average day.

1.2.2 Visa

Visa is the world's leading payment card and financial service card brands. It is operated by Visa International Service Association. It is a blue, white and gold logo with the word Visa placed in between a blue stripe and gold stripe, with a white background. There are approximately 21 000 financial institutions around the world, which are Visa card members that handles settlement of over 100 million transactions a day (Visa, 2005). These transactions are processed by the Visa Net system.



1.3 OBJECTIVES OF RESEARCH

In this research, there are several objectives:

- i. To analyze the check digit scheme of a credit card number, known as the IBM scheme.
- ii. To determine the factors in the application of Mod 10 as a mathematical method to verify the validity of a credit card number.
- iii. To develop a C program to verify the validity of a credit card number.
- iv. To understand the implementation of a four-digit Personal Identification Number (PIN) in the United Kingdom as an extended security feature to verify a cardholder and to authorize a card transaction.

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1.4 SCOPE OF RESEARCH

The scope of research focuses mainly on the application of the check digit scheme, known as the IBM scheme, which is used by the two major credit card brands in Malaysia, MasterCard and Visa, in checking the validity of a credit card number.



CHAPTER 2

LITERATURE REVIEW

2.1. HISTORY OF CREDIT CARDS

The use of credit cards originated in the United States. Around 1914, oil companies and large department stores began to experiment with credit cards by allowing some customers to charge purchases (cited in Mcleod, 1979). The first credit card was issued in that year by an oil company, General Petroleum Corporation of California, now known as Mobil Oil. The card was issued to the employees and selected customers for local gasoline purchases made at those businesses.

In 1951, Franklin National Bank in New York issued the first modern credit card (Auriemma & Coley, 1992). In 1959, a new credit card lending system was transformed, when the Bank of America in California developed a national bank plan, known as Bank Americard. The development of this credit card system sparked visions of success, principally in economics of scale, a proper monthly interest charge to the cardholder, and a discount on the value of the paper transaction paid by the shopkeeper or merchant (Edward, 1985).



By 1961, Bank Americard had achieved a total of one million cardholders amounting to \$ 75 million and by 1967, this total had increased to 2.7 million cardholders amounting to \$ 335 million. The successfulness of the Bank Americard system led to the establishment of the Bank Americard Service Corporation (BSC) in 1966. It engaged in the licensing of other banks in the United States. In that same year, the system was licensed to Barclays Bank in Great Britain, United Kingdom. Barclays Bank is the first British bank to launch credit cards in the United Kingdom which began issuing Barclaycard, linked to Bank Americard (now known as Visa) in 1966 (Edwards, 1997).

The licensing developed into an international bank card system, now known as Visa International. Visa International comprised members from financial and non-financial institutions. The system's success was based on economics of scale, an appropriate monthly interest charge to the cardholder, and a discount on the value of the paper transaction paid by the merchant.

In July 1970, the Bank of America sold the Bank Americard Service Corporation which became the independent National Bank Americard Incorporated (NBI). As a result of the increased use of Bank Americard, IBANCO, a separable non-stock membership international organization was incorporated in 1974. The Bank of America realized a need to make the card known internationally. In 1976, the name Bank Americard was changed to the word Visa as a result of its understanding and international acceptance. A year later, NBI became Visa USA and IBANCO became VISA International.



Due to the rapid growth of Bank Americard, major Californian banks such as Wells Fargo Bank, United California Bank, Bank of California and Crocker National Bank formed the California Bank Card Association as a non-profit organization issuing a common bank credit card (Drury & Ferrier, 1984). The California Bank Card Association purchased the rights to use the 'Master Charge' name and the card design was obtained from the First National Bank of Louisville, Kentucky, United States.

As a result of the growth of the Master Charge credit card programme, a group of enterprising bankers established the Interbank Card Association (ICA). The group comprised agent banks representing 17 financial institutions of seven bankcard schemes that issued the Master Charge credit card. The association's functions were to manage interchange functions of authorization, clearing and settlement between different regions of the country. The cards of the association were indicated by a small white "i" symbol within a black symbol located at the lower right-hand corner of the card (Drury & Ferrier, 1984).

In 1969, the ICA acquired the exclusive rights to the 'Master Charge' name, logo, service marks and the interlocking circles design from the California Bank Card Association (now known as the Western States Bankcard Association (WBSA)). By 1970, over 5000 financial institutions, with almost 36 million cardholders in almost every state in the United States were promoting the 'Master Charge' name. In 1979, due to the international expansion, ICA decided to change the name and design of 'Master Charge'. By 1983, 'Master Charge' was renamed 'MasterCard'.



2.2 CREDIT CARDS

A photo sample of the front look and back look of a MasterCard with a chip is shown in Photo 2.1 and Photo 2.2.



Photo 2.1 The front look of a MasterCard with a chip

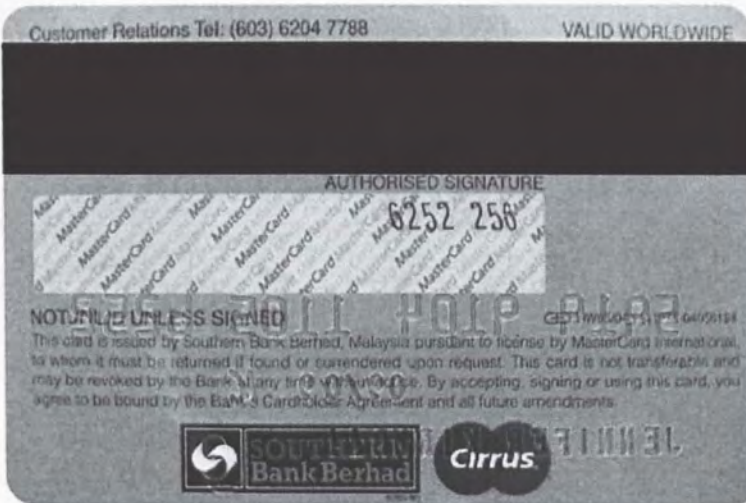


Photo 2.2 The back look of a MasterCard

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