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WANG SIEW ENN

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I hereby declare that this dissertation contains my original research work. Source of findings reviewed herein have been duly acknowledged.

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1. **SUPERVISOR**

   (Assoc. Prof. Dr. Ho Chong Mun)

2. **EXAMINER 1**

   (Mr. Victor Tiong Kung Ming)

3. **EXAMINER 2**

   (Dr. Aini Janteng)

4. **DEAN**

   (Supt/KS. Assoc. Prof. Dr. Shariff A. Kadir S. Omang)

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WANG SIEW ENN
HS 2005-1068

ABSTRACT

This dissertation was intended to offer an explanation of the Platonic Solids, Archimedean Solids and Kepler Poinsot Solids,at once simple and practical but not too speculative. There are some characteristic and properties have been observed and discussed. Every polyhedron follows the Euler's Formula and has its dual polyhedron. However, there is an exception of Euler's Formula for Kepler Poinsot Solids. All the Platonic Solids, Archimedean Solids and Kepler Poinsot solids can be described in an easier form like schlafli symbol and vertices configuration. The tessellation of polyhedron shows its arrangement of polygonal faces. Archimedean can be formed by truncation and snubbing process of Platonic Solids. On the other hand, Kepler Poinsot can be constructed by stellations of Platonic Solids. Some regular polyhedra share the common vertex arrangement or same edge arrangement.

ABSTRAK

Kajian ini menyelidik tentang Pepejal Platonik, Pepejal Archimedean, dan Pepejal Kepler Poinsot. Ciri-ciri am telah dikaji dalam kajian ini. Setiap polihedron mematuhi hukum Euler dan mempunyai polyhedron dual. Namun begitu, terdapat pengecualian untuk kes Pepejal Kepler Poinsot dalam hukum Euler. Semua polihedron seragam dan semiseragam dibentang dalam bentuk yang lebih mudah iaitu simbol schlafli dan configurasi puncak. *Tessellation* bagi polyhedron menggambarkan susunan untuk suatu polyhedron. Pepejal Archimedean boleh dihasilkan dengan kaedah *truncation* dan *snubbing*. Di samping itu, Pepejal Kepler Poinsot boleh dibentuk melalui *stellation* bagi Pepejal Platonik. Beberapa polyhedron seragam menikmati susunan puncak yang sama atau susunan tepi yang sama.

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=	equal
+	addition
-	substraction
Σ	summation

CHAPTER 1

INTRODUCTION

1.1 What Is Polyhedron?

A polyhedron is a closed, three-dimensional version of polygon. It is composed of polygons connected at their edge in order to enclose space. In another word, it is a geometric object with flat surfaces and straight edges. The plural of polyhedron is polyhedra or polyhedrons (Alexander, 2003).

The word polyhedron is derived from Greek in which poly- means "many" and -edron means "base", "seat" or "face". A polyhedron consists of vertex (vertices in plural), edges and faces as shown in Figure 1.1 below.

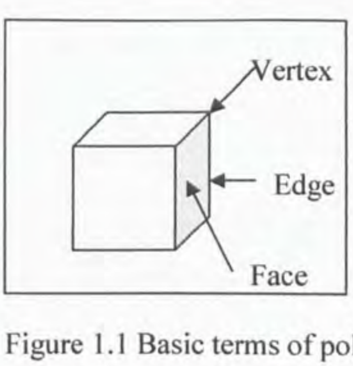


Figure 1.1 Basic terms of polyhedron

A face of the polyhedron is each of its polygonal sides whereas an edge is a line segment where two faces meet, it is formed by the sides of two faces. A point called vertex is formed when few edges and faces are meet. This modified definition is given by Cromwell (1997) in his book "Polyhedra".

According to Henderson & Daina (1996) in "Experiencing Geometry Euclidean and Non-Euclidean with History", there are few types of angles involved in polyhedron. Plane angle is the angle at the corner of a polygonal face. Dihedral angle is the angle created by two adjacent faces which have two sides joined to form an edge; it can be measured by the angle of two rays that lie in the planes which have a vertex on the edge and perpendicular to the edge as shown in Figure 1.2. On the other hand, solid angle is the portion of interior of a polyhedron at a vertex which is surrounded by three or more plane angles as shown in Figure 1.3 below.

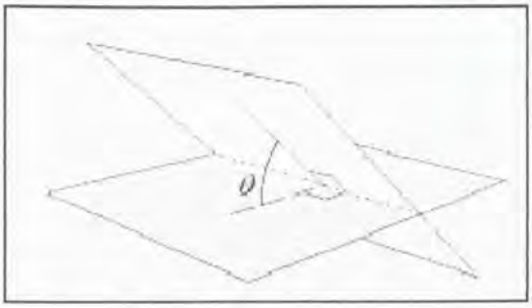


Figure 1.2 Dihedral angle

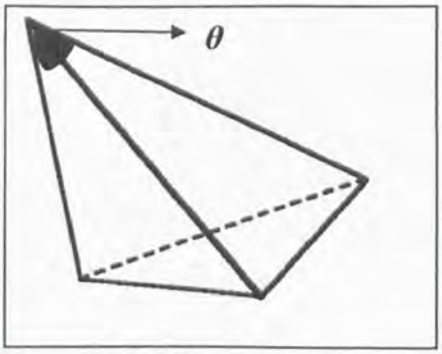


Figure 1.3 Solid angle

There are few common properties shared by polyhedra.

i. Naming of polyhedra

Naming of polyhedra is almost similar to the naming of polygon but more complicated. Naming of polyhedra based on Classical Greek according to its number of faces as shown in Table 1.1 below. Though, there are some special polyhedra have their own name such as Szilassi polyhedron.

Table 1.1 Naming polyhedra

Number of faces	Polyhedron
4	Tetrahedron
5	Pentahedron
6	Hexahedron
7	Heptahedron
8	Octahedron
9	Nonahedron
10	Decahedron
11	Undecahedron
12	Dodecahedron
14	Tetradecahedron
20	Icosahedron
24	Icositetrahedron
30	Tricontahedron

Table 1.1 Naming polyhedra (continued)

32	Icosidodecahedron
60	Hexecontahedron
90	enneacontahedron

ii. Edges

Edges have two important features that it just joins two faces and two vertices with the exception of those complex polyhedra which constructed in unitary three-space with six dimensions.

iii. Euler Characteristic

All the polyhedron obey Euler Characteristic with V-E+F= 2, where V is the number of vertices, E is the number of edges, and F represents the number of faces. According to Malkevitch, this formula was discovered in around 1750 by Euler, and first proven by Legendre in 1794.

iv. Duality

There exists a dual polyhedron for every polyhedron. Dual polyhedron is the polyhedron that associates with another polyhedron with one faces correspond to the vertices of another and vice versa. Thus, the number of faces of one polyhedron is equal to the number of vertices of its dual polyhedron.

v. Vertex figure

A vertex figure which consisting of the vertices which join to it can be defined from every vertex. In another word, vertex figure is formed when a polyhedron is truncated. Vertex figure is illustrated in Figure 1.4 below.

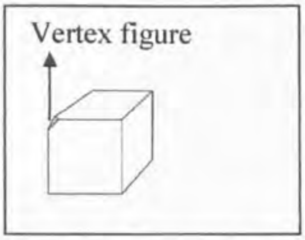


Figure 1.4 Vertex figure

As indicated by Alexandrov (2005), a polyhedron is said to be convex if it composed of many planar polygons so that it is possible for a polygon to pass from one to another by polygons which having common sides of segment of sides and the whole figure lies on one side of the plane of each constituent polygons.

A polyhedron is said to be regular if its faces are made up of regular polygons that have equal sides and angles. It means that every face consists of same number of vertices and vice versa.

1.1.1 Platonic Solids

Another name for platonic solids is regular polyhedron. It is convex regular polyhedra. Platonic solids are perfectly regular solids with congruent faces which made up of identical polygons. It follows few of conditions where all its faces are identical, all its dihedral angles are the same and all its sides are equal.

There are only five platonic solids exist. The five platonic solids are tetrahedron, cube, octahedron, dodecahedron, and icosahedron as shown in Figure 1.5 below.

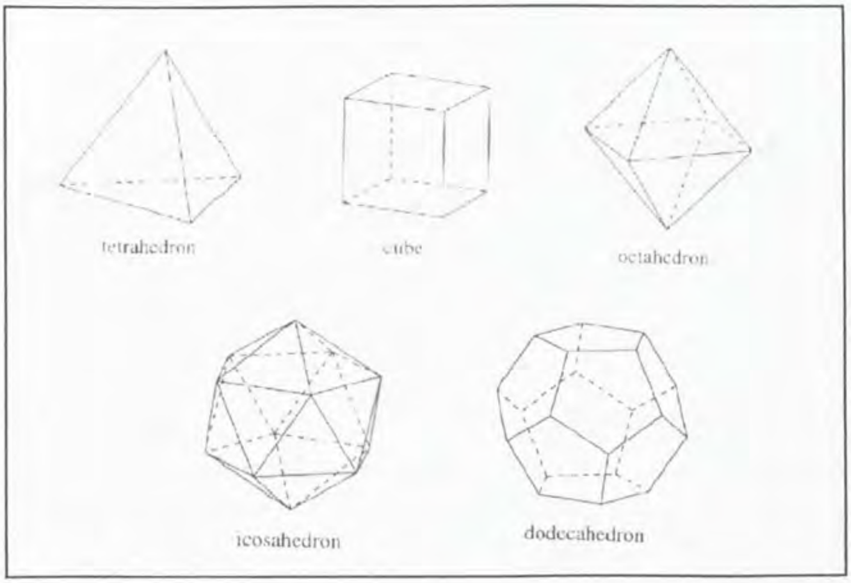


Figure 1.5 Platonic Solids

1.1.2 Archimedean Solids

Sometimes are called Archimedean Polyhedra which are convex semi regular polyhedra. Archimedean polyhedra are composed of equilateral and equiangular but not similar polygons. Its faces might be made up of different types of polygons but its vertices are identical. Every vertex of Archimedean Solids is congruent which means the faces must be arranged in the same order around each vertex.

There are thirteen Archimedean Solids which named truncated tetrahedron, truncated cube, truncated octahedron, truncated dodecahedron, truncated icosahedron, cuboctahedron, icosidodecahedron, rhombicuboctahedron, rhombicosidodecahedron, great rhombicuboctahedron, great rhombicosidodecahedron, snub cube, and snub dodecahedron. All these Archimedean solids are shown in the Figure 1.6 below.

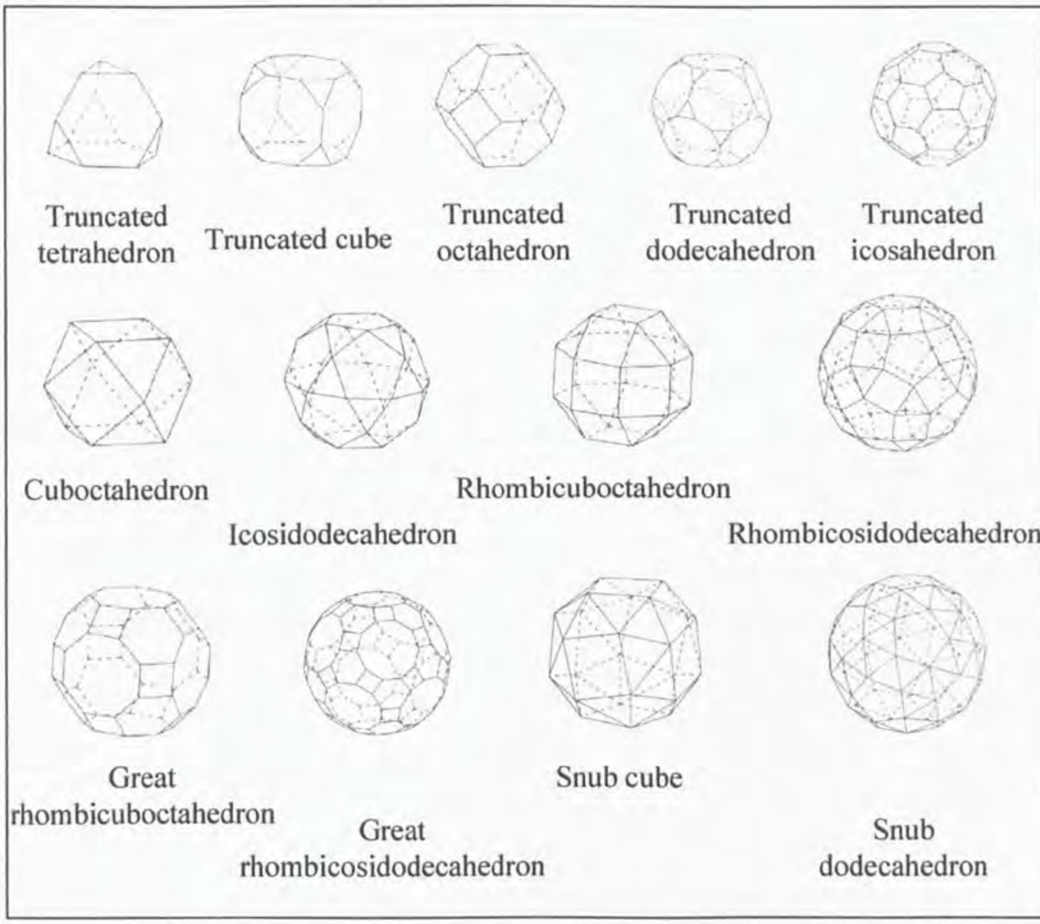
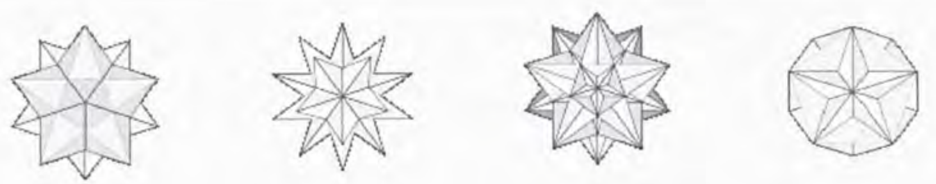


Figure 1.6 Archimedean Solids

1.1.3 Kepler Poinsot Solids

There exist four types of regular non-convex polyhedra or can be called regular concave polyhedra which known as Kepler Poinsot Solids. They are polyhedra that made up of regular concave polygons with intersecting planes. The number of faces meeting at each vertex is the same.

Small stellated dodecahedron and great stellated dodecahedron are known as Kepler solids which discovered by astronomer Johannes Kepler whereas the great icosahedron and great dodecahedron discovered by French mathematician, Louis Poinsot, named as Poinsot solids. The four Kepler Poinsot solids are shown in the Figure 1.7 below.



Small stellated dodecahedron | Great stellated dodecahedron | Great icosahedron | Great dodecahedron

Figure 1.7 Kepler Poinsot Solids

1.2 Historical Background

Carved stones from the Neolithic time about 1000 years before Plato have been discovered in Scotland. They are oldest known regular polyhedra. Several of the regular polyhedra and the semiregular cuboctahedron were known in Babylon, Egypt, India and China. For example, the ancient pyramids at Giza, Egypt was built over 4500 years ago. The geometry was developed during the golden period of ancient Greek culture.

The earliest written record of these polyhedra shapes are came from classical Greek. After that, Islamic scholars continued to make it advances, for example in the tenth century Abu'l Wafa describes two and three dimensional construction. Meanwhile in China, the formula for the volume of a truncated pyramid was derived by Liu Hui by dissecting pyramid into a central square-based prism.

. In the Ranaissance, the Greek literature was translated to Latin from Arabic and later from Greek. Pierro della Francesca (1410-1492) studied the five regular platonic solids and six of the semi regular Archimedean polyhedra. Leon Battista Alberti (1404-1472) and Pierro della Francesca introduced the theory of the perspective according to Vitruvius (50 B.C).

Luca Pacioli (1445-1517) described the Platonic solids and six of Archimedes polyhedra. On the other hand Leonardo da Vinci drew the polyhedra in Pacioli's book "De Divina Proportione" but he did not know the collection by Pappus and thought that there were an unlimited number of semi regular polyhedra. Albrecht Dürer designed the polygons of the regular polyhedra and nine of the semiregular polyhedra onto a plane in the form of "nets" (flat patterns) in Underweisung year 1525. (Cromwell, 1997)

1.2.1 Platonic Solids

The regular polyhedra were attributed by ancient Greek and studied by a group of Greek mathematicians under the supervision of Pythagoreans as far back as 500BC.

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