

**BUILDING A C++ PROGRAMME WITH CUBIC
SPLINE ESTIMATION METHOD TO ESTIMATE
SHELL SHARE PRICES**

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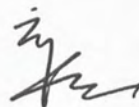


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DECLARATION

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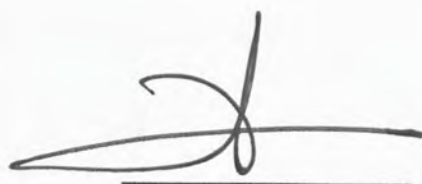


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ABSTRAK

Objektif kajian tersebut adalah menggunakan *C++ programming* untuk membentuk satu program dengan menganggar nilai yang hilang, kaedah *cubic spline* digunakan dalam kajian tersebut. Program penganggaran dengan kaedah *cubic spline* adalah digunakan untuk menganggar kehilangan nilai dalam harga saham harian Syarikat Shell. Program penganggaran dengan kaedah *cubic spline* adalah dikhaskan untuk menganggar kehilangan indeks dalam masa yang berturut-turut. Program penganggaran dengan kaedah *cubic spline* mengintegrasikan dengan fungsi *auto-scanning*, dan juga fungsi *auto-printing*. Program tersebut bermula dengan menganalisis data dalam “data.txt”, dan buat pengiraan dengan menempatkan nilai kepada 12x12, 16x16, atau 20x20 matriks. Selepas menyongsangkan matriks tersebut, fungsi *spline* didedahkan, justerunya, program tersebut akan mengira nilai-nilai di antara data yang menggunakan fungsi *spline*. Selepas kehilangan nilai dikira, langkah terakhir untuk program tersebut adalah menghasilkan data-data bersama dengan hampir semua nilai yang dikira dalam satu fail yang bernama “output”. Langkah-langkah tersebut akan diulangi hingga hujung fail “data.txt”



ABSTRACT

The purpose of this research is to use C++ programming language to build a programme to do estimation on missing values, by using cubic spline method. The cubic spline estimation programme will be used to estimate the missing values in daily share prices of SHELL Company. The cubic spline estimation programme is designed to estimate missing indexes that corresponding time series. The cubic spline estimation programme integrated with auto-scanning function, as well as auto-printing function. The programme will start with scanning the data in “data.txt”, and do the calculations by locating corresponding values into 12x12, 16x16, or 20x20 matrixes. The next computation step is to invert the 12x12, 16x16, or 20x20 matrixes. After inverting the matrixes, the spline functions are now revealed, the programme will hence compute the missing values among the data by using the spline functions. After computing the missing values, the last step of the programme is to print the data together with all the newly calculated values into a file named “output.txt”. These steps are repeated until the end of the “data.txt” file.



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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

This paper concern about several methods that will be used to estimate missing values that exist in our share market, and writing a program to do the estimation on missing values.

From what Chia and Soh (1991) had said, the concept of share market is introduced by companies in order to collect capitals from share buyers. Companies gain their capitals from buyers, hence companies can have more investment plans or programs to gain profits, and buyers will be rewarded by interest from the shares that they are holding.

According to Chia and Soh (1991), at Malaysia, our stock exchange activities are monitored by Bursa Malaysia Berhad. Bursa Malaysia Berhad is an exchange holding company, listed on the Main Board of Bursa Malaysia Securities. Bursa Malaysia Berhad



is a non-governmental and non-profit organization. They operate a fully integrated exchange offering the complete range of exchange-related services, including trading, clearing, settlement and depository services, and also provide information services related to the Malaysian securities market. Bursa Malaysia is committed to maintaining an efficient, secure and active trading market for local and global investors.

Malaysia have three types of share markets, that are main board (or we call first board), second board, and MESDAQ. Main Board is usually for larger capitalized companies, the Second Board for medium sized companies or the MESDAQ Market for high growth and technology companies. MESDAQ market is always having higher risk than others.

Chia and Soh (1991) also stated that, the beginnings of today's stock exchange was on 21 March of 1960, ten firms in Singapore and seven firms in Malaysia set up the Malayan Stock Exchange to provide proper trading rooms where shares changed hands based on share prices shown on a board. Two trading rooms were set up, one in Singapore and another in Kuala Lumpur. These were linked by direct telephone lines in 1962.

The stock exchange was renamed the Stock Exchange of Malaysia in 1964 after Singapore joined the Federation of Malaysia. Later on, the stock exchange change name to the Stock Exchange of Malaysia and Singapore, when Singapore left Malaysia. Another split came in May 1973 when the Government decided to discontinue the



interchangeability of the Malaysian ringgit and the Singapore dollar. The exchanges were renamed the Stock Exchange of Singapore (SES) and the Kuala Lumpur Stock Exchange (KLSE). On April 2004, Kuala Lumpur Stock Exchange had been recognized by government and launched Securities Commission to supervise the development of the securities industry in Malaysia. Kuala Lumpur Stock Exchange was renamed again as Bursa Malaysia. One year after renamed, Bursa Malaysia has become one of the listed organizations on the Main Board of Bursa Malaysia Securities Berhad on 18 March.

According to Chia and Soh (1991), from the legal perspective, the demutualization essentially entailed the conversion from a not-for-profit "mutual" entity limited by the guarantee of its members into a company limited by shares. However, from the business strategy perspective, the demutualization, supported by business transformation initiatives, is intended to further enhance its corporate, organizational and governance structures.

1.1.1 Shell Refining Company

From what Annual Report of Shell Company 2006 stated, Shell is a global group of energy and petrochemical companies, which active in more than 130 countries and territories and employs 108,000 people worldwide. Shell is an energy company which explores for, produces and trades in a range of energy resources, they explore for and produce oil and gas and create essential products, such as fuels and petrochemicals. Shell also has a broad portfolio of hydrogen, biofuels, wind and solar power interests. Shell



provides consultancy and technical services as well as research and development expertise to the energy industry.

According to Shell Company Annual Report 2006, at the year of 1833, Marcus Samuel, a young shopkeeper selling antiques in London East End, decided to expend his merchandise to include oriental shells. Shells were hugely popular at this time and the decision transformed his shop into a highly successful import/export business. Little did he realize, in an industrial age powered by coal, this was the foundation for a business that would be at the very heart of the oil industry.

From the Shell Company Annual Report 2006, Shell has five core businesses: Exploration and Production, Gas and Power, Refining and Marketing, Chemicals, and Trading/Shipping, and operates in more than 140 countries. Shell's primary business is the management of a vertically integrated oil company. The development of technical and commercial expertise in all the stage of this vertical integration from the initial search for oil through its harvesting, transportation, refining and finally trading and marketing established the core competencies on which the Group was founded. Similar competencies were required for natural gas, which has become one of the most important businesses in which Shell is involved, and which contributes a significant proportion of the company's profits.

Based on the Shell Company Annual Report on 2006, while Shell in the past, the vertically integrated business model gave significant economies of scale and provided



Shell with the opportunity to establish barriers to entry both geographically and on a more global scale, this has been less a possibility in more recent time. As a result although the vertical integration remain there is much less interdependence between businesses and each is now charge with being a self-supporting independent business without cross subsidies from other parts of the business chain.

1.2 PROBLEMS OF MISSING VALUES

Based on research by Morgan (2005), there are so many methods are available for analyzing financial data to help decision makers. But in practical, we always have some problems with the data that is available, and most of the methods that will be used for analyzing are so sensitive to the values of the data, so we might have to deal with bad data before modeling or calculation take place. Improving the quality of the data can improve the quality of results.

Problems of data that we usually encountered:

- i. No values have been input.
- ii. Impossible values have been input.
- iii. Inconsistence values have been input.
- iv. Unlikely values have been input.



From what Morgan (2005) said, we usually have two types of missing values, first is structural missing and observational missing. Structural missing will relate to values that we are not expecting to be there (Just like the share prices changes will not be available of weekend, and holidays). Observational missing is just the data or value has gone astray.

1.3 MISSING VALUES IN OUR SHARE MARKETS

According to Kok and Goh (1995), in our daily life, share buying and selling activities are held on everyday in two sessions, first session is 9.00 am to 12.00pm, and second session is from 2.00pm to 5.00pm, except public holidays and weekends. So there will be missing data in share prices. These missing data is a problem for researcher who study the price trend of a curtain share, so these missing values in data should be estimated. We have about 52 weeks in a year, there are at least 104 weekends that the share market does not trade, and it does not include those special celebrating festivals. Out of 365, there are more than 104 missing data, there are at least 28.49% of data are missing, so estimating these data are essential.

Referred from Giordano, *et al.* (1997) there are a few methods to estimate, or find out the missing values, such as, markov change, multiple regressions, and cubic spline. Basically the cubic spline method is used to estimate missing values by using interpolation; we can even call cubic spline as spline interpolation.



A cubic spline is a spline construct of piecewise third-order polynomials which pass through few control points, the second derivative of each polynomial at the end points is zero, this attribute provide few equations to the system, hence the mathematical model can be solved.

1.4 PROBLEM TO SOLVE

Kok and Goh (1995) claimed that, one of the problem exist in our share market Malaysia is that we always have too much of missing values in share prices comparing with other countries. Because Bursa Malaysia does not open for trading while holidays, and total public holidays of Malaysia are more than other foreign countries. Some of the foreign countries share markets are open for trading even at holidays, this makes that missing values that exist in our share prices occupied a large portion every year, compare to other countries.

From what Morgan (2005) said, when we are doing forecasting, we are predicting the future, by using old historical data, in order to have a better forecasting model, what we need is to have better historical data. In other words, we should estimate the missing values in our data, to provide a better, a more-rounded and higher amount of historical data, to help out in forecasting. Hence estimating missing values in our data is an important issue.



According to Morgan (2005), missing values exist sometime not only due to “does not exist of certain value”, but also can happen to be the data files are corrupted or being destroyed in any accident. So estimating the damaged data is one of the methods to recover the lost data.

1.5 ESTIMATING MISSING VALUE BY USING HIGH-LEVEL PROGRAMMING LANGUAGE

There are too many missing values in our share market, estimating all the values one by one might not be a time-wise choice, but those missing values are essential for researchers to work the trend out.

Hence a high-level programming language (such as C++, FORTRAN, PASCAL, etc) can help in this aspect, building a programme with method cubic spline to calculate the missing value can help researchers to estimate a more accuracy missing value, by using lesser time. So a high accuracy missing value estimating programme is very important, it can be a great savior for researchers in terms of time and even cost saving.

As stated by Mayo and Martin (1995), the computer language FORTRAN is basically quite identical with C, they both have problem with graph drawing, the calculating programmes like MATLAB, MAPLE, SPSS, can conduct the complicated calculation easily, and also displaying a nice time-series graph, but these programme cannot be used to build a user-friendly missing value estimating programme. C is the



computer language that this paper will use to build the programme, because this is the simplest programme with multi-function.

1.6 OBJECTIVE OF STUDY

- i. Using Cubic Spline method to find out or estimate the missing values of SHELL share prices from 2000 to 2007.
- ii. Build a user-friendly programme which helps to estimate missing indexes, by using cubic spline method.
- iii. To integrate auto-estimating functions into the programme, so the programme can estimate missing values effectively, saving time and cost for users.
- iv. To help out trend-researchers to find out clearer trend of a certain share price, hence researchers can do more accurate forecasting.
- v. To estimate a better data, repair broken or incomplete data by using Cubic Spline estimation programme.

1.7 LIMITATION

Since we are using the cubic spline method to estimate the missing value, so the accuracy of the estimated value might be questionable. There are also few choices for cubic spline method. We can use different numbers of points to do the estimation, and the result might



differ from each others. For example: we can choose to use a four points cubic spline approach or five points approach to do the estimation.

There is a minimum requirement in numbers of data; we cannot estimate missing values by using insufficient numbers of data. For cubic spline with four points approach, the minimal points we need for each variable is four, we cannot do the estimation with less than four points of data.

The cubic spline method is only suitable for missing values estimating, but not forecasting or predicting any index. Cubic spline can work well in estimating the missing values among some data, and the estimated values are reasonable and acceptable, but this method is only suitable for estimation, cubic spline method cannot work when we come to forecasting or predicting future.

1.8 RATIONALE OF STUDY

We always encounter with missing values in our data. When there are too much missing values in a set of data, due to the data are physically gone astral, or unavailable of certain data, we will need to estimate the missing values, in order to have a better, clearer data for further uses.

By Morgan (2005), all the trend researchers, and forecasters are doing their forecast, or prediction based on the previous data, so historical data is an important issue

for those people who are doing researches to find out the best model to forecast. Estimating missing values definitely will help them to have more data, to do better, and more accuracy predictions.

Estimating missing values by using programming would be a great savior to people who deal with massive amounts of missing data. Estimating by using programme, can help in saving in term of cost, time, and even man-power. So programming estimation is a time-wise choice for a person who faces tons of missing values everyday.



CHAPTER 2

LITERATURE REVIEW

2.1 MISSING VALUES

In the paper by Morgan (2005), missing values are always a big trouble for researchers and also decision makers. Bad data such as missing values or inconsistency values should be repaired or replaced with a better and more likely data. This is because the sophisticated methods are always very sensitive to the values of data, a good model could not be constructed by using incomplete data.

According to Morgan (2005), to deal with missing values or bad data, the first is to find the observation that is most similar to the one with the missing value; this is called the donor. If there is more than one possible donor, then the donor used can be selected in different ways, either by choosing one randomly or by selecting the first in the list. In these approaches the skill is in selecting the variables to match the recipient and donor.



The second approach is to use models to predict the missing value. The fundamental approach is:

- i. Fit a model.
- ii. Use the model to predict the missing value.

2.2 EXPECTATION-MAXIMIZATION ALGORITHM

In 2004, Koji, *et al.* purposed that, an expectation-maximization algorithm is used in statistics for finding maximum likelihood estimates of parameters in probabilistic models, where the model depends on unobserved latent variables. Expectation-Maximization alternates between performing an expectation step, which computes an expectation of the likelihood by including the latent variables as if they were observed, and a maximization step, which computes the maximum likelihood estimates of the parameters by maximizing the expected likelihood found on the Expectation step. The parameters found on the Maximization step are then used to begin another Expectation step, and the process is repeated.

Based on the research by Koji, *et al.* (2004), estimating missing data in a matrix is often done with methods, such as the Expectation-Maximization Algorithm, using the existing data in that matrix. However, if auxiliary data that is related to the missing measurements is available, it can help to estimate the missing values. This paper also presents latent variable approaches that exploit an auxiliary data information matrix, as well as the data matrix itself. The use of auxiliary information is most useful when the

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